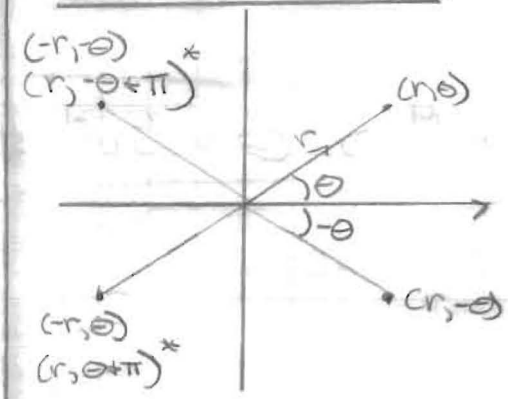
10.3 - Polar Coordinates

TOP BOARD

Basics - Multiple



Conversions

① $(r,\theta) \to (x,y)$

$x = r\cos\theta$

$y = r\sin\theta$

② $(x,y) \to (r,\theta)$

$r^2 = x^2 + y^2$

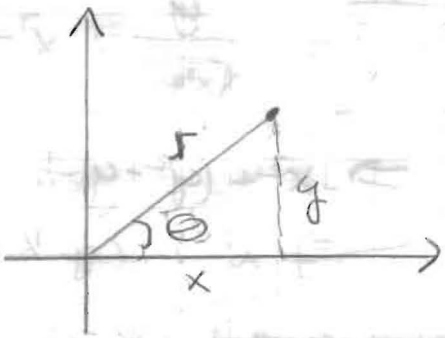
$\tan\theta = \frac{y}{x}$

(I) Basics - (Above)

Example ⇒ Plot $(1, \frac{\pi}{2})$, $(-1, \pi)$, $(1, 0)$, $(\sqrt{2}, \pi/4)$, $(-\sqrt{2}, 3\pi/4)$, $(\sqrt{2}, 3\pi/4)$, etc.

(II) Conversions

(A) $(r,\theta) \Rightarrow (x,y)$



$\cos\theta = \frac{x}{r} \Rightarrow \boxed{x = r\cos\theta}$

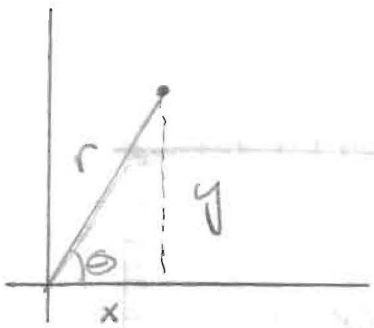
$\sin\theta = \frac{y}{r} \Rightarrow \boxed{y = r\sin\theta}$

$r = 3,\ \theta = \pi/3$

$x = 3\cos\frac{\pi}{3} = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

$y = 3\sin\frac{\pi}{3} = 3\frac{\sqrt{3}}{2} = \frac{3}{2}\sqrt{3}$

(B) $(x,y) \to (r,\theta)$



$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

Example

$= (3,4) \Rightarrow (r,\theta)$

$\Rightarrow r^2 = 3^2 + 4^2 \Rightarrow r^2 = 9+16 = 25$

$\boxed{r = 5}$

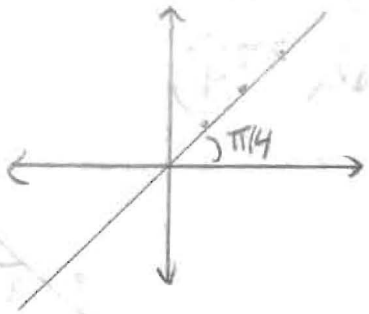
$\Rightarrow \tan\theta = \frac{4}{3} \Rightarrow \boxed{\theta = \tan^{-1}\frac{4}{3}}$

(III) Graphing Polar Equations

$r = f(\theta)$

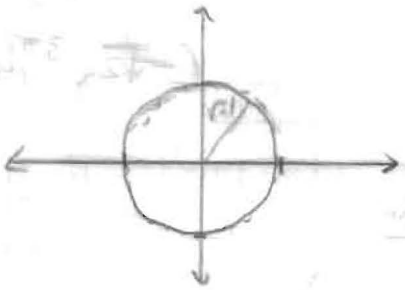
Example

(e.g. ① $\theta = \pi/4$



② Example

e.g. $r = 1$



$x = r\sin\theta \Rightarrow \sin\theta = \frac{y}{r}$

③ Example

$r = \sin\theta \to$ convert $\Rightarrow$

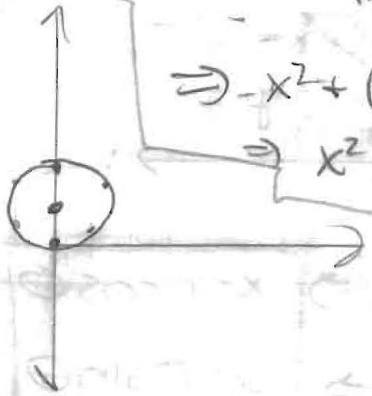
$\sqrt{x^2+y^2} = \frac{y}{\sqrt{x^2+y^2}} \Rightarrow x^2+y^2 = y$

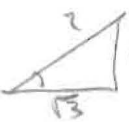
$\Rightarrow -x^2 + (y^2 - y + \frac{1}{4}) = 0 + \frac{1}{4}$

$\Rightarrow x^2 + (y-\frac{1}{2})^2 = \frac{1}{4}$

$x = 0,\ y = \frac{1}{2}$

$r = \frac{1}{2}$





θ	r
0	
π/6	1/2
π/3	√3/2
π/2	1
⋮	

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(IV) Using Maple - polarplot(f(θ))