Section 11.1 Sequences (#42)

Definitions	Therems About Convergence

I. DEFINITION: SEQUENCE

A list of numbers written in a definite order

$$a_1, a_2, a_3, \ldots a_n$$

$\Rightarrow a_1 =$ 1st number, $a_2 =$ second number, etc...

Notation: $\{a_1, a_2, a_3, \ldots\}$, $\{a_n\}$, $\{a_n\}_{n=1}^{\infty}$

$$a_n = \frac{n}{n+1} \Rightarrow \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \frac{n}{n+1}, \ldots \right\}$$

Sequence is like function $f(x)$ where $f(x)$ is only evaluated at natural numbers

$$f(x) = \frac{x}{x+1} \Rightarrow \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4} \ldots \frac{x}{x+1} \ldots \right\} \text{ for } x = 1, 2, 3, 4 \ldots$$

II Definition – Limit

① Sequence has a "limit" if $\lim_{n\to\infty} a_n = L$ (a finit #)

② if $\lim_{n\to\infty} a_n$ exists then a_n is "convergent"

(i.e. $\lim_{n\to\infty} \frac{n}{n+1} = 1 \Rightarrow$ convergent)

③ if $\lim_{n\to\infty} a_n$ DNE, than "divergent"

i.e. $\lim_{n\to\infty} \frac{n^2}{n+1} = \infty$ (DNE) $\Rightarrow$ divergent

- $\lim_{n\to\infty} (-1)^n$ = either 1 or -1 (DNE) divergent

III Definition: Increasing/Decreasing

① if $a_{n+1} > a_n$ for $\forall\ n>0 \Rightarrow$ "increasing" $\Rightarrow$ i.e. $a_n = \frac{1}{n}$

② if $a_{n+1} < a_n$ for $\forall\ n>0 \Rightarrow$ "decreasing" $\Rightarrow$ i.e. $a_n = \ln(n)$

③ in either case it is "monotonic"

IV Definition: Bounded Above/Below

① "Bounded Above" if $a_n \leq M$ for $\forall n$

② "Bounded Below" if $a_n \geq m$ for $\forall n$

i.e. $a_n = \frac{1}{n}$ bounded above by 1

bounded below by 0

③ "bounded sequence" $\Rightarrow$ bounded above & below

V Therems about Convergence

$\boxed{1}$ If $\lim_{x \to \infty} f(x) = L$ and $a_n = f(n)$
then $\lim_{n \to \infty} a_n = L$

Example
$a_n = \frac{n}{2n+1}$

$\boxed{2}$ If $\lim_{n \to \infty} |a_n| = 0$, then $\lim_{n \to \infty} a_n = 0 \Rightarrow$ Example $a_n = (-1)^n \left(\frac{1}{n^2}\right)$

$\boxed{3}$ If $a_n = r^n$, $\lim_{n \to \infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \\ \text{DNE} & \text{for } \forall \text{ other values} \end{cases}$

Example ~~lim~~ $a_n = \left(\frac{1}{2}\right)^n$

$\boxed{4}$ If a_n is (a) bounded
(b) monotonic
$\Rightarrow a_n$ = convergent ! $\Rightarrow$ Example ① $\{0, 2, 0, 2, 0 \ldots\}$
② $a_n = \frac{1}{n}$

$\boxed{5}$ - Squeeze Therem -

If $a_n < b_n < c_n$ then $\lim_{n \to \infty} a_n \le \lim_{n \to \infty} b_n \le \lim_{n \to \infty} c_n$

Example: $\frac{\sin(n)}{n}$

$\boxed{6}$ Limit Laws (Page 566)