Section 11.2 - Series

#4/3

Definitions	Therems
Seq	Geo Series
Series	Therom
Diverges	
Converges	
The	

I. Series vs. Sequence

a) "Sequence" - a list of numbers in a specific order.

i.e. $a_n = 1, 2, 3, 4, 5 \ldots\ldots n$

b) "Series" - a list of numbers that result from a sum of a sequence

"partial sums"

$$\begin{cases} S_1 = a_1 \\ S_2 = a_1 + a_2 \\ S_3 = a_1 + a_2 + a_3 \\ S_n = \sum_{i=1}^{n} a_n \end{cases}$$

i.e. → if $a_n = 1, 2, 3, \ldots n$

$S_1 = 1$

$S_2 = 3$

$S_3 = 6$

$\vdots$

$S_n = \sum_{n=1}^{n} n = \frac{(n)(n-1)}{2}$

II) Series "Converges" if $\sum_{i=1}^{\infty} a_n = L$ (a finite #)

Series "Diverges" if it does not converge

Examples (

$a_n = n$ $\boxed{\text{"Go 1"}}$ <

$a_n = \left(\frac{1}{2}\right)^{n-1}$ $\boxed{\text{"Go 2"}}$

$a_n = (-2)^n$ $\boxed{\text{"Go -3"}}$

II Theroms

$\boxed{1}$ If $\sum_{i=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$

$\boxed{\text{"Go 2"}}$

⇒ IS THIS TRUE?

If $\lim_{n\to\infty} a_n = 0$ then $\sum_{i=1}^{\infty} a_n$ is convergent?

$\boxed{\text{"Go 4"}}$ $\sum_{n=1}^{\infty} \frac{1}{n}$ (20, 50, 100, 250) ..

⇒ answer no!!

However

$\boxed{2}$ If ⓐ $\lim_{n\to\infty} a_n$ DNE or ⓑ $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{i=1}^{\infty} a_n$ divergent } "Divergent Test"

$\sum_{i=1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^n\right)$ $\lim_{n\to\infty} \left(1 - \left(\frac{1}{2}\right)^n\right) = 1$

⇒ ↳ divergent $\boxed{\text{go5}}$

(III) Geometric Series

Definition $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$

Therom : (1) Geometric Series is convergent if $|r| < 1$

(and)

(2) $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

Example $\boxed{\text{"906"}}$

(1) $\sum_{n=1}^{\infty} \frac{4}{3^n} = \sum_{n=1}^{\infty} 4\left(\frac{1}{3^n}\right) = \sum_{n=1}^{\infty} 4\left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{4}{3}\left(\frac{1}{3}\right)^{n-1}$

$a = \frac{4}{3} \quad r = \frac{1}{3} \Rightarrow \sum_{n=1}^{\infty} a_n = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{4}{2} = 2$

$\boxed{\text{"907"}}$

(2) $5 + \frac{10}{3} + \frac{20}{9} + \dots$

$= 5\left(1 + \frac{2}{3} + \frac{4}{9} \dots\right) = \sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1} = \frac{5}{1-\frac{2}{3}} = \frac{5}{\frac{1}{3}} = 15$

$\boxed{\text{"908"}}$

(3) $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27}$

$\Rightarrow 5\left(1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} \dots\right) = \sum_{n=1}^{\infty} 5\left(-\frac{2}{3}\right)^{n-1} = \frac{5}{1-\left(-\frac{2}{3}\right)}$

$\Rightarrow \frac{5}{\frac{5}{3}} = \boxed{3}$