Section 11.5 - Lecture 1 - Alternating Series (#44)

(I) Definition

Alternating Series - a series whose terms are alternately + and -

i.e. $S_n = 1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} \ldots + (-1)^{n-1}\left(\frac{1}{n}\right)$

$\Rightarrow S = \sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n}\right)$

i.e. $S_n = -\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \ldots (-1)^n \frac{n}{n+1}$

$\Rightarrow S = \sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)$

(II) Alternating Series Test

If $\sum_{n=1}^{n} (-1)^n b_n = b_1 - b_2 + b_3 - b_4 \ldots b_n > 0$

and ① $b_{n+1} < b_n$ for all n

② $\lim_{n \to \infty} b_n = 0$

Then Series is convergent

(III) Alternating Series Estimation Theorem

If $S = \sum (-1)^{n-1} b_n$ and

(a) $b_{n+1} < b_n$ (2) $\lim_{n\to\infty} b_n = 0$

then $|R_n| = |S - S_n| \leq b_{n+1}$

R_n: remainder

Example

$S_n \sum_{n=1}^{n} (-1)^{n-1} \frac{1}{n}$

(a) $\frac{1}{n+1} < \frac{1}{n}$ (b) $\lim_{n\to\infty} \frac{1}{n} = 0$ } therefore convergent

$S_{50} = .6832$ $|R_{50}| \leq .019608$

$S_{100} = .68817$ $|R_{100}| \leq .009901$

$S_{250} = .68762$ $|R_{250}| \leq .0066225$

} got 1

Example

$S_n = \sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n!}\right)$

Got 2

$S_{50} = .6321$ $|R_{50}| < 6.647 \times 10^{-47}$

By the way ...

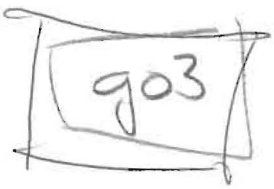
Example

$$\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n^3}\right)$$

⇒ find sum ~~to~~ so Remainder $|R| \leq .0001$

$\left(\frac{1}{n+1}\right)^3 \leq .0001 \Rightarrow \frac{1}{n+1} \leq .046416 \Rightarrow 21.5443 \leq n+1$

$\Rightarrow n \geq 20.5443 \Rightarrow$ let $n=21$



IV

Teams – (598)
3, 4, 5, 6, 7, 8

12, 13, 14, 17, 18, 15, 16