Section 11.6 - Ratio Test

Absolute Convergence / Ratio Test

TOP BOARD

① Definition of Absolute Convergence	(III) Theorem - P-Series
(II) Abs Convergent theorem	(IV) Ratio TEST

Do P-Series 1ST

(I) Definition:

① let $S_n = \sum_{n=1}^{n} a_n$, if we consider

$\sum_{n=1}^{n} |a_n| = |a_1| + |a_2| + |a_3| \ldots$

② If $\sum_{n=1}^{\infty} |a_n|$ is convergent, then $\sum a_n$ is "absolutely convergent"

Example:

Consider: $\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n^2}\right) = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} \ldots$

$\sum_{n=1}^{\infty} \left|(-1)^{n-1}\left(\frac{1}{n^2}\right)\right| = \sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right) = 1 + \frac{1}{2} + \frac{1}{9} \ldots$

$\boxed{\text{Go 4}}$ (III) Explain P-series

$\sum_{n=1}^{\infty} \frac{1}{x^p}$ converges if $p > 1$

(II) Theorem

If $\sum_{n=1}^{\infty} |a_n|$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent

$\Rightarrow$ But not necessarily the other way around.

$\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$ is convergent $\left\{ \begin{array}{l} \lim_{n\to\infty} \frac{1}{n} = 0 \\ \frac{1}{n+1} < \frac{1}{n} \end{array} \right.$

$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ is not! $\{$ p series

(III) P Series $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p}$ $\left\{ \begin{array}{ll} p > 1 & \text{convergent} \\ p \le 1 & \text{divergent} \end{array} \right.$

(IV) Ratio Test

If ① $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ $\left\{ \begin{array}{ll} < 1 & \sum a_n \text{ abs convergent} \Rightarrow \text{converg.} \\ > 1 & \sum_{n=1}^{\infty} a_n \text{ divergent} \\ = 1 & \text{no conclusion} \end{array} \right.$

(V) Boardwork - Ratio Test

(599) # 19, 20, 21, 22, 24, 25