11.8 - Power Series

#46

Top Board

① Power Series Def	④ Ratio Test
② Rad Conv	
③ Int Conv.	

(I) Power Series Definition

ⓐ — $\sum_{n=1}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 \ldots$

↑ coefficients

ⓑ — MORE GENERALLY

$\sum_{n=1}^{\infty} C_n (x-a)^n$ (Power Series centered on "a")

ⓒ Aside: → $\sum_{n=1}^{\infty} x^n$ is a geometric series whose sum is $\frac{1}{1-x}$ when $-1 < x < 1$

II Example

When Does $\sum_{n=1}^{\infty} n^2 x^n$ converge

Ratio test $\lim_{n\to\infty} \left| \frac{(n+1)^2 (x^{n+1})}{n^2 (x^n)} \right|$

$= \lim_{n\to\infty} \left| \frac{n+1}{n} x \right| = |x| < 1$

$\Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

↑ Radius of Convergence

III Definition:

① "Radius of Convergence": a number R, such that the power series $\sum_{n=1}^{\infty} c_n (x-a)^n$ converges when $|x-a| < R$

② "Interval of Convergence" - all values of x for which $\sum_{n=0}^{\infty} c_n (x-a)^n$ converges

i.e. $|x-a| < R \Rightarrow -R < x-a < R$

$\Rightarrow a-R < x < a+R$

Example

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n\,2^n}$$

Ratio Test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} \frac{(x+2)^{n+1}}{(n+1)(2^{n+1})}}{(-1)^n \frac{(x+2)^n}{n\,2^n}} \right|$

$$= \lim_{n\to\infty} \left| \frac{(-1)\, n\, (2^n)(x+2)^{n+1}}{(n+1)(2^{n+1})(x+2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n}{n+1} \frac{1}{2} x+2 \right| = \left| \frac{x+2}{2} \right| < 1$$

↑ for convergence

$\Rightarrow |x+2| < 2$ ← R $\Rightarrow -2 < x+2 < 2$

$\Rightarrow -4 < x \leq 0$

↑ I

GO 1 ⇒ compares for $x = -4.1, -3.9, -2, -.1, .1, .5$

II Board (604) #3 -17