Section 11.9 - Functions as Power Series

Top Board

- Must Important Power Series	Theorem
$\frac{1}{1-x} = 1+x+x^2+x^3\dots = \sum_{n=1}^{\infty} x^{n-1} = \sum_{n=0}^{\infty} x^n$	$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$
$\Rightarrow \lvert x\rvert<1$	$f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$
	$\int f(x)dx = C + \sum_{n=0}^{\infty} \frac{c_n(x-a)^{n+1}}{n+1}$

I Consider

Note same

$$\left.\begin{array}{c}\sum_{n=1}^{\infty} x^{n-1} \\ \sum_{n=0}^{\infty} x^n\end{array}\right\} = 1+x+x^2+\dots$$

geometric series w/ $a=1$, $r=x$

$\therefore \sum_{n=0}^{\infty} x^n = \frac{a}{1-r} = \frac{1}{1-x}$

$\therefore \frac{1}{1-x} = 1+x+x^2+\dots \Rightarrow |x|<1$

$\Rightarrow$ Show this is true by sythetic division

$\Rightarrow$ (G01) - pauses (up to 6 terms)

$\sum = 1$, $\sum = 1+x$, $\sum = 1+x+x^2$, ... $\sum = 1+x+x^2+x^3+x^4+x^5$

(II) Power Series for

(a) using $\frac{1}{1-x}$

$$\frac{1}{1-x^2} = 1 + x^2 + (x^2)^2 + (x^2)^3 \dots \Rightarrow |x|<1$$

$$1 + x^2 + x^4 + x^6 + \dots = \sum_{n=0}^{1} (x)^{2n}$$

(b) $\Rightarrow$ sythetic division

(III) Power Series for

(a) $\frac{1}{1+x^2} \Rightarrow$ write as $\frac{1}{1-(-x^2)}$

$$= 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 \dots$$

$$= 1 - x^2 + x^4 - x^6 \dots = \sum_{n=0}^{1} (-1)^n x^{2n}$$

(b) $\Rightarrow$ sythetic division ...

(c) go2

(IV) Therom - IF power series $\sum^{1} c_n (x-a)^n$ has a Radius of $R>0$ ~~and f(x~~

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots c_n(x-a)^n$$

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 \dots nc_n(x-a)^{n-1}$$

$$= \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

$$\int f(x)dx = c_0 x + \frac{1}{2}c_1(x-a)^2 + \frac{1}{3}(x-a)^3 \dots \frac{1}{n+1}c_n(x-a)^{n+1} + C$$

$$= C + \sum_{n=1}^{\infty} \frac{c_n(x-a)^{n+1}}{n+1}$$

(V) Arctan(x) = $\tan^{-1}(x) = \int \frac{1}{1+x^2}\,dx$

$$= \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx = \sum_{x=0}^{\infty} (-1)^n \left(\frac{1}{2n+1}\right) x^{2n+1}$$

$$= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 \ldots$$

(VI) Board work - ex $\boxed{\frac{1}{16+4x^2}}$

Power Series: $\frac{1}{1-x}$, $\frac{1}{1-x^3}$, $\frac{1}{1+9x^2}$, $\frac{1}{4+x^2}$,

$\frac{1}{5-x}$, $\frac{x}{4x+1}$

Find General Expression