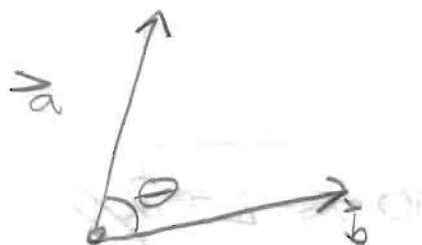


Section 12.3 - Dot Products (Day 1)

I. Definition

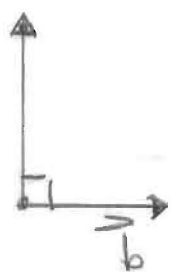


$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\theta)$$

(a "scalar")

II. ORTHOGONALITY OF Vectors

$\vec{a} \cdot \vec{b} = 0$, vectors are orthogonal



i.e. $|\vec{a}| |\vec{b}| \cos \theta$

$\theta = \frac{\pi}{2}$

III. Algebraic Dot Product

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

(a scalar)

IV Example

Find Angle between

$$\vec{a} = \langle 2, 2, -1 \rangle \quad \vec{b} = \langle 5, -3, 2 \rangle$$

$$\Rightarrow \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = (2)(5) + (2)(-3) + (-1)(2) = 10 - 6 - 2 = 2 //$$

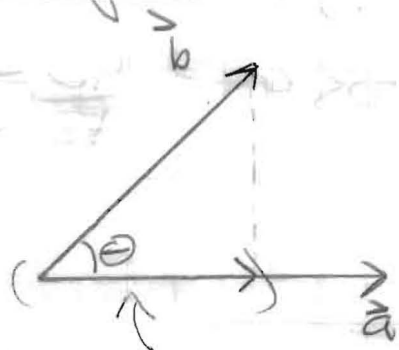
$$|\vec{a}| = (4 + 4 + 1)^{1/2} = 3 \quad |\vec{b}| = (25 + 9 + 4)^{1/2} = (38)^{1/2}$$

$$\Rightarrow \cos \theta = \frac{2}{(3)(38)^{1/2}} \approx 0.081 \Rightarrow \theta = \underline{\underline{83.79^\circ}}$$

Section 12.3 (Day 2)

(I) Properties of Dot Product (664)

(II) Projections



$$|\vec{b}| \cos \theta = |\vec{b}| \frac{\vec{a} \cdot \vec{b}}{|\vec{b}| |\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

(a) \Rightarrow Scalar Projection of \vec{b} onto \vec{a}

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \quad (\text{scalar}) - \text{length of projection}$$

(b) \Rightarrow Vector Projection of \vec{b} onto \vec{a}

$$\text{proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \left(\frac{\vec{a}}{|\vec{a}|} \right) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

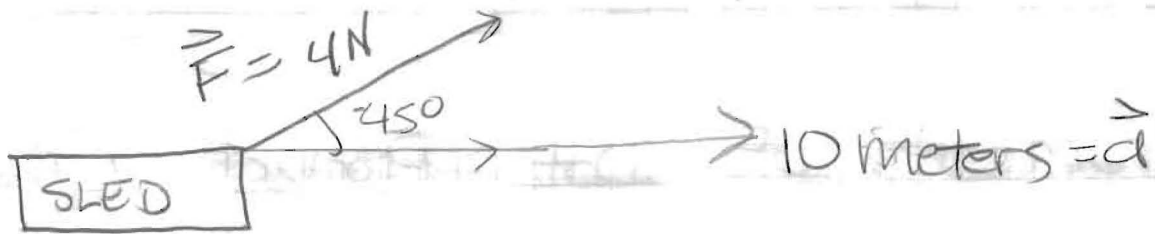
(III) Example: $\vec{a} = \langle 1, 2, 3 \rangle$ $\vec{b} = \langle 3, 2, 1 \rangle$

$$\text{comp}_{\vec{a}} \vec{b} = \frac{3+4+3}{(1+4+9)^{1/2}} = \frac{10}{\sqrt{14}}$$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{10}{\sqrt{14}} \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}} = \frac{\langle 10, 20, 30 \rangle}{14} = \left\langle \frac{5}{7}, \frac{10}{7}, \frac{15}{7} \right\rangle$$

(IV)

Example



$$\bar{w} = (|\vec{F}| \cos \theta) (|\vec{d}|) = |F||d| \cos(45^\circ)$$

$$w = \vec{F} \cdot \vec{d} = (4)(10) \cos 45 = \frac{40}{\sqrt{2}} \text{ J} = \underline{\underline{20\sqrt{2} \text{ J}}}$$