Section 12.4 – Cross Product (Day 1) (Day-2)

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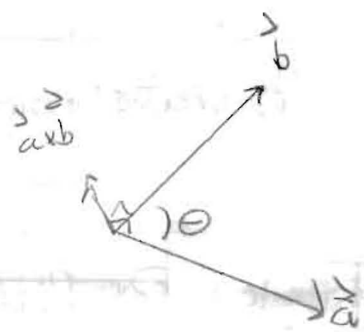
Definition	Properties
$\vec{a}\times\vec{b} = (\lvert a\rvert\lvert b\rvert \sin\theta)\,\vec{n}$	① $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$
→ $\vec{n}$ is a vector perpendicular to both $\vec{a}$ + $\vec{b}$	② $(c\vec{a})\times\vec{b} = c(\vec{a}\times\vec{b}) = \vec{a}\times(c\vec{b})$
→ $\vec{n}$ is a unit vector	③ $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}+\vec{c}$
→ right hand rule	④ $(\vec{a}+\vec{b})\times\vec{c} = \vec{a}\times\vec{c} + \vec{b}\times\vec{c}$

(II) Definition

$$\vec{a}\times\vec{b} = (\lvert a\rvert\lvert b\rvert \sin\theta)\,\vec{n}$$

⇒ RIGHT HAND RULE



(III) Properties of Cross Product (see above)

(IV) Cross Product in Component Form

$\vec{a} = \langle a_1, a_2, a_3\rangle$

$\vec{b} = \langle b_1, b_2, b_3\rangle$

$$\vec{a}\times\vec{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1\rangle$$

⇒ using determinants

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \vec{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= \vec{i}(a_2b_3 - a_3b_2) - \vec{j}(a_1b_3 - a_3b_1) + \vec{k}(a_1b_2 - a_2b_1)$$

$$= \vec{i}(a_2b_3 - a_3b_2) + \vec{j}(a_3b_1 - a_1b_3) + \vec{k}(a_1b_2 - a_2b_1)$$

$$\langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_3 - a_2b_1 \rangle \checkmark\checkmark$$

(V) Scaler Triple Product

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

volume of "parallelepiped"

(VI) Vector Triple Product

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

(OPTIONAL)

(VII) $\boxed{\text{CROSS PRODUCT APPLET}}$

(VIII) Examples

(IX) TORQUE $|\tau| = |r \times F|$ Example 1 (668)