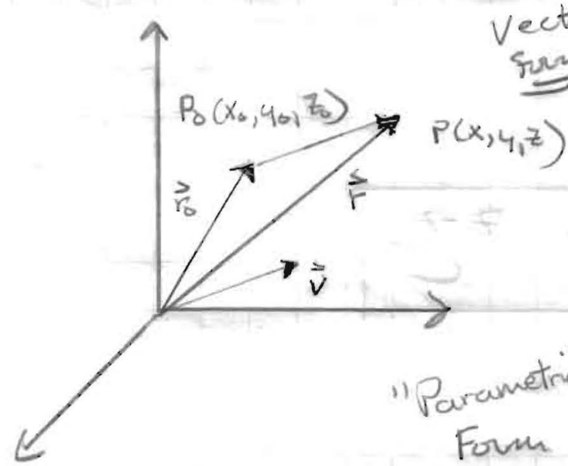
Section 12.5 Equations of Lines (I)



Vector form: (1) $\vec{r} = \vec{r}_0 + \vec{v}\,t$

or if $\vec{v} = \langle a, b, c \rangle$

"Parametric" Form:

(2) $\langle x, y, z \rangle = \langle x_0 + ta,\ y_0 + tb,\ z_0 + tc \rangle$

or

(3) $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$

(I) Vector Form

i.e. $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle = \langle 1, 2, 3 \rangle$

$\vec{r} = \langle x, y, z \rangle = \langle 3, 1, 4 \rangle$

$\vec{v} = \vec{r} - \vec{r}_0 = \langle 2, -1, 1 \rangle$ "$= \langle a, b, c \rangle$"

$\therefore\ \vec{r} = \langle 1, 2, 3 \rangle + \langle 2, -1, 1 \rangle t$

II Parametric Form

$\langle x, y, z \rangle = \langle 1 + 2t,\ 2 - t,\ 3 + t \rangle$

or

$x = 1 + 2t$
$y = 2 - t$
$z = 3 + t$

III. Symmetric Form

Solve for t

$\Rightarrow \quad t = \frac{x-1}{t} = 2-y = z-3$

or in general:

$$\boxed{\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}}$$

(IV) Show that

$\langle x, y, z\rangle = \langle 1, -2, 4\rangle + \langle 1, 3, -1\rangle t$

$\langle x, y, z\rangle = \langle 0, 3, -3\rangle + \langle 2, 1, 4\rangle s$

(a) are not parallel $\Rightarrow \langle 1, 3, -1\rangle \neq \langle 2, 1, 4\rangle$

(b) are skew $\Rightarrow$ i.e. have no point of intersection.

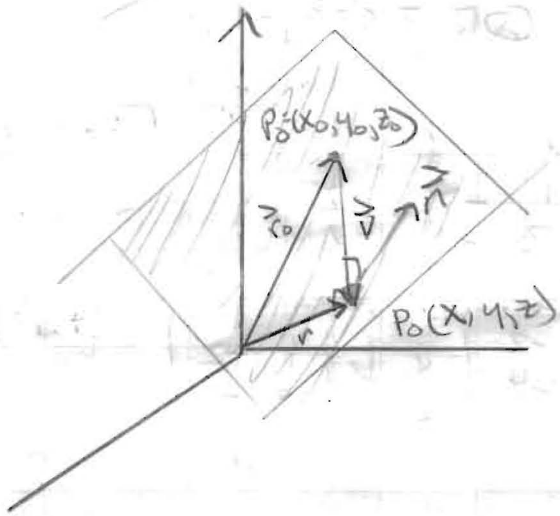
If Intersect, $\exists$ s, t such that

$$\left.\begin{aligned} 1+t &= 2s \\ -2+3t &= 3+s \\ 4-t &= -3+s \end{aligned}\right\} \text{ solve } \begin{aligned} &\Rightarrow s = \tfrac{1}{2} + \tfrac{1}{2}t \\ &\Rightarrow -2+3t = 3 + \tfrac{1}{2} + \tfrac{1}{2}t \\ &\Rightarrow \tfrac{5}{2}t = \tfrac{11}{2} \Rightarrow t = \tfrac{11}{5}, s = \tfrac{8}{5} \end{aligned}$$

but $= 4 - \frac{11}{5} \neq \frac{9}{5} \neq -3 + \frac{8}{5} = -\frac{7}{5}$

$\therefore$ no intersection exists

Section 12.5 - Equation of Planes (II)



<u>Scaler Equation</u>

$ax + by + cz + d = 0$

where $\vec{n} = \langle a, b, c \rangle$

$P_0 = \langle x_0, y_0, z_0 \rangle$

$d = -\vec{n} \cdot \vec{P}_0$

(I) <u>Scaler Equation of a PLANE</u> w/

$$\left.\begin{array}{l} \vec{r}_0 = (x_0, y_0, z_0) \\ \vec{r} = (x, y, z) \end{array}\right\} \vec{V} = \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$n = \langle a, b, c \rangle \; \} \; \vec{n} \cdot \vec{V} = 0$$

$$\Rightarrow \vec{n} \cdot \vec{v} = 0 \Rightarrow ax - ax_0 + by - by_0 + cz - cz_0 = 0$$

$$\Rightarrow \text{let } d = -ax_0 - by_0 - cz_0$$

$$\therefore \; ax + by + cz + d = 0$$

<u>Example</u> $\Rightarrow$ Equation of Plane w/ $P_0 = \langle 2, 4, -1 \rangle$ and $\vec{n} = \langle 2, 3, 4 \rangle$

$$\Rightarrow 2(x-2) + 3(y-4) + 4(z+1) = 0$$
$$\Rightarrow 2x + 3y + 4z - 12 = 0$$

Example

Find Equation of Plane through

$P(1,2,3)$ $Q(2,0,1)$ $R(-1,-1,-1)$

$\vec{PQ} = \langle 1,-2,-2\rangle$ $\vec{QR} = \langle -3,-1,-2\rangle$

$$\Rightarrow n = \vec{PQ} \times \vec{QR} = \begin{vmatrix} i & j & k \\ 1 & -2 & -2 \\ -3 & -1 & -2 \end{vmatrix}$$

$$= i\begin{vmatrix} -2 & -2 \\ -1 & -2 \end{vmatrix} \cancel{+} j\begin{vmatrix} 1 & -2 \\ -3 & -2 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ -3 & -1 \end{vmatrix}$$

$$= 2i + 8j - 7z$$

$$\therefore\ 2(x-1) + 8(y-2) - 7(z-3) = 0$$

$$\Rightarrow 2x + 8y \cancel{+} 7z + 3 = 0$$

(II) Distance Between Point (x_1, y_1, z_1) and Plane $ax + by + cz + d = 0$

$$= \boxed{\frac{|ax_1 + cy_1 + dz_1 + d|}{\sqrt{a^2+b^2+c^2}}}$$