Section 13.2 Derivatives & Integrals of Vector Functions

Top Board

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

$$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}$$

$$\int \vec{r}(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle$$

unit tangent vector $= \vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

Example

$$\vec{r}(t) = 2\cos(t)\vec{i} + 2\sin(t)\vec{j} + 3t\vec{k}$$

$$\vec{r}\,'(t) = -2\sin(t)\vec{i} + 2\cos(t)\vec{j} + 3\vec{k}$$

$$|\vec{r}\,'(t)| = [4\sin^2(t) + 4\cos^2(t) + 9]^{1/2}$$

$$= [4+9]^{1/2} = \sqrt{13}$$

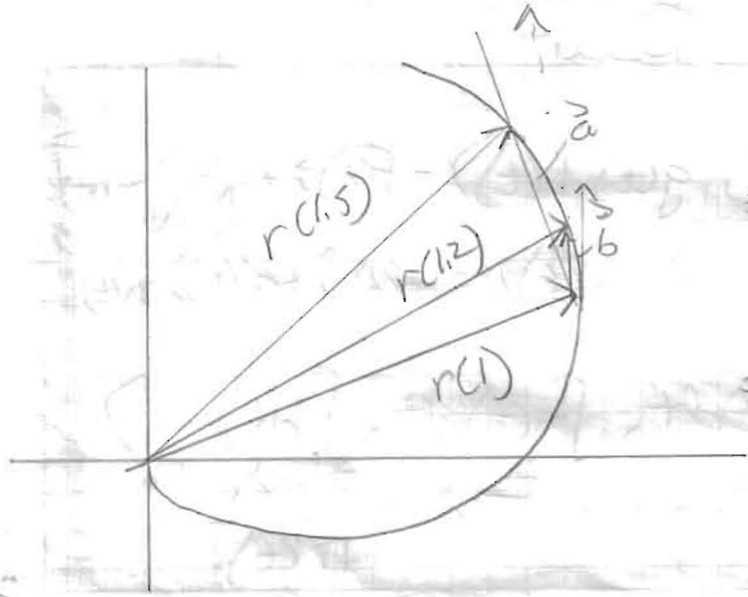
$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -2\sin(t), 2\cos(t), 3 \rangle}{\sqrt{13}}$$

$$\int \vec{r}(t)\,dt = (2\sin(t) + C_1)\vec{i} + (-2\cos(t) + C_2)\vec{j} + \left(\tfrac{3}{2}t^2 + C_3\right)\vec{k}$$

(II) Definition of Derivative

$$\vec{r}\,'(t) = \frac{d}{dt}\vec{r}(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) + \vec{r}(t)}{h}$$

analysis



$r'(1)$?

$\Rightarrow h = .5 \Rightarrow \frac{\vec{a}}{.5} = 2\vec{a}$

$h = .2 \Rightarrow \frac{\vec{b}}{.2} = 5\vec{b}$

$\Rightarrow h = 0 \Rightarrow$ tangent to curve

*** All rules of derivatives apply ***

(III) Example (Board)

$\vec{r}(t) = (2\sin t)\,\vec{i} + (5\cos(t))\,\vec{j} + t^3\,\vec{k}$, Find $r'(\frac{\pi}{2})$

$\vec{r}\,'(t) = \langle 2\cos t, -5\sin t, 3t^2 \rangle$

$\Rightarrow r'(\pi/2) = \langle 0, -5, \frac{3\pi^2}{4} \rangle$

Example

Find $\int \vec{r}(t)\,dt = \vec{R}(t) \Rightarrow \vec{R}(0) = \langle 0, 3, 2 \rangle$

$\int \vec{r}(t)\,dt = \langle -2\cos(t) + c_1, 5\sin(t) + c_2, \frac{1}{4}t^4 + c_3 \rangle = R(t)$

$\Rightarrow R(0) = \langle -2 + c_1, c_2, c_3 \rangle = \langle 0, 3, 2 \rangle$

$\Rightarrow c_1 = 2, c_2 = 3, c_3 = 2$

(IV) MAPLE EXAMPLE $\Rightarrow$ #24