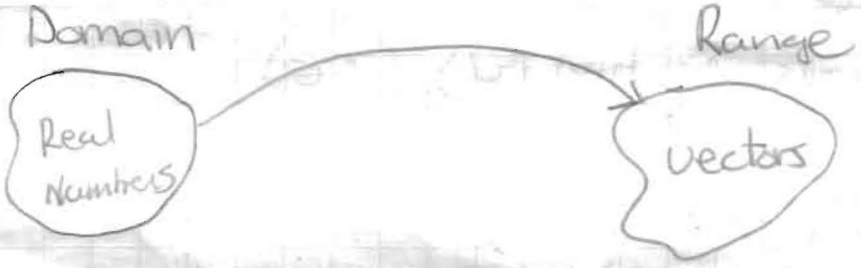
Section 13.1 – Vector Functions & Space Curves

⇒ Project parametrics on board

(I): Vector Value Function –

a) 

b) used to describe curves in space

c) form: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$

"component functions"

(II) Example

$\vec{r}(t) = \langle t^4, \ln(5-t), \sqrt{t+1} \rangle$

Domain: t^4 : $t \in \mathbb{R}$
$\ln(5-t)$: $t < 5$
$\sqrt{t+1}$: $t \geq -1$
} $-1 \leq t < 5$

Range: $1 \leq t^4 \leq 625$, $-\infty < \ln(5-t) \leq \ln(6)$, $0 \leq \sqrt{t+1} \leq \sqrt{6}$

(III) Limits

$$\lim_{t\to a} \vec{r}(t) = \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \right\rangle$$

(IV) Continuity

continuous: $\lim_{t\to a} r(t) = r(a)$

(V) Example:

$$\vec{r}(t) = \left\langle \frac{\sin(t)}{t}, \frac{t^2+t}{t}, \sqrt{t} \right\rangle$$

$$\lim_{t\to 0} \vec{r}(t) = \left\langle \lim_{t\to 0} \frac{\sin(t)}{t}, \lim_{t\to 0} \frac{t^2+t}{2}, \lim_{t\to 0} \sqrt{t} \right\rangle = \langle 1, 0, 0 \rangle$$

(VI) Space Curve = plot of vector functio

i.e. for $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

plot $\left.\begin{array}{l} x = f(t) \\ y = g(t) \\ z = h(t) \end{array}\right\}$ – Parametric Equations

(VII) Example - Sketch $\langle \cos(t), \sin(t), t \rangle$

(VIII) Using Computers (Maple)

spacecurve ([f(t), g(t), h(t)], $a \le t \le b$)

i.e. $\vec{r}(t) = \langle \sin t/t, t^2+t/t, \sqrt{t} \rangle$

$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$

$\vec{r}(t) = \langle 4+\sin 20t, 4+\cos 2t, \cos 20t \rangle$