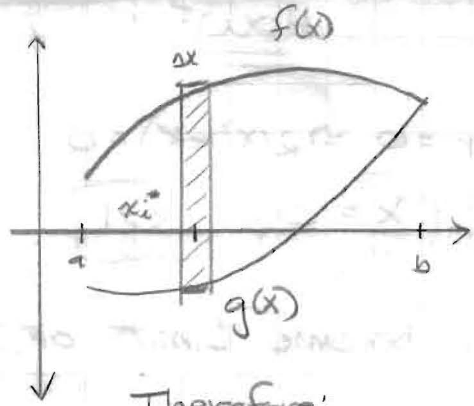
SECTION 6.1 - MORE ABOUT AREAS

HW: (452) 1, 4, 11, 14, 19, 21, 25

TOP BOARD



height: $f(x_i^*) - g(x_i^*)$
width: Δx

$A_i = [f(x_i^*) - g(x_i^*)]\Delta x$

$A_{TOT} = \sum_{i=1}^{n} A_i$

<u>Therefore:</u>

$$\lim_{n\to\infty} A_{TOT} = \lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x$$
$$= \int_a^b [f(x) - g(x)]\, \underline{dx}$$

I <u>EXAMPLE</u>

let $f(x) = \exp(x)$ and $g(x) = x$ for $0 \le x \le 1$

Find area between curves

* - { MATLAB - go! for $n = 10, 50, 100$

<u>Analytically:</u>

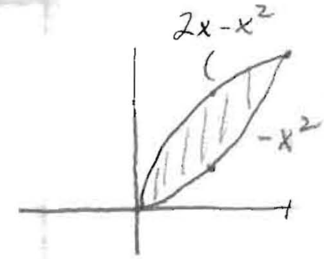
$$\int_0^1 (e^x - x)\,dx = e^x - \tfrac{1}{2}x^2 \Big|_0^1 = (e - \tfrac{1}{2}) - (1 - 0)$$
$$= e - 3/2 \approx \underline{1.21828}$$

II Example 2

Find AREA BETWEEN REGIONS "ENCLOSED" BY

$y = 2x - x^2 \quad , \quad y = x^2$

FIRST FIND INTERSECTION HOW?



$2x - x^2 = x^2 \Rightarrow 2x^2 - 2x = 0 \Rightarrow 2(x^2 - x) = 0$

$\Rightarrow x(x-1) = 0 \Longrightarrow \boxed{x = 0, \quad x = 1}$

Become LIMITS OF INTEGRATION

Numerical
GO 2 $\Rightarrow n = 10, 50, 100 \quad A \to 1/3$

ANALYTIC

$$\int_0^1 [(2x - x^2) - x^2]\,dx = \int_0^1 (2x - 2x^2)\,dx = x - \tfrac{2}{3}x^3 \Big|_0^1$$

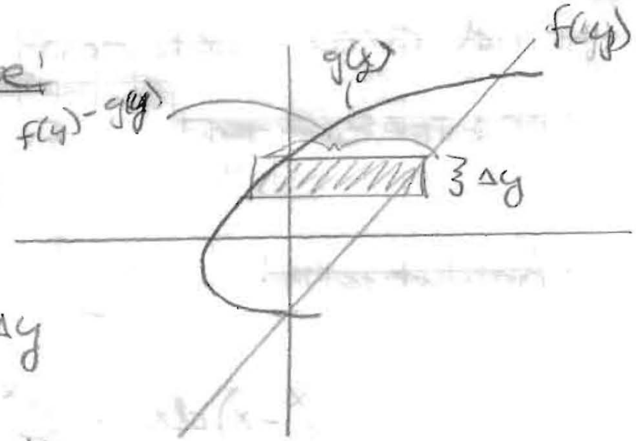
$$= (1 - \tfrac{2}{3}) - (0 - 0) = 1/3$$

III What if instead we have:

$x = f(y)$

$x = g(y)$

$A_i = f(y_i) - g(y_i)\,\Delta y$



$$A_{TOT} = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} [f(y_i) - g(y_i)]\,\Delta y$$

$$\lim_{n\to\infty} A_{TOT} = \lim_{n\to\infty} \sum_{i=1}^{n} [f(y_i) - g(y_i)]\,\Delta y$$

$$= \int_{y=a}^{y=b} [f(y) - g(y)]\,\Delta y$$

IV Example 3

Find Area Enclosed by

$$\left.\begin{aligned} y &= x-1 \\ y^2 &= 2x+6 \end{aligned}\right\}$$ Find points of intersection

$(x-1)^2 = 2x+6 \Rightarrow x^2 - 2x + 1 = 2x+6 \Rightarrow x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0 \Rightarrow x = -1,\ x = 5$
$y = -2,\ y = 4$

$\Rightarrow$ plot: let $x = y+1$
$x = \frac{1}{2}(y^2-6)$

$\Rightarrow$ Numerical g05 - for plot   Area $\approx$

$\Rightarrow$ Analytic

$$\int_{-2}^{4} \left[(y+1) - \tfrac{1}{2}(y^2-6)\right] dy = \int_{-2}^{4} \left(y + 1 - \tfrac{1}{2}y^2 + 3\right) dy$$

$$= \int_{-2}^{4} \left(-\tfrac{1}{2}y^2 + y + 4\right) dy = -\tfrac{1}{6}y^3 + \tfrac{1}{2}y^2 + 4y \Big|_{-2}^{4}$$

$$= \left[-\tfrac{1}{6}(64) + \tfrac{1}{2}(16) + 16\right] - \left[-\tfrac{1}{6}(-8) + \tfrac{1}{2}(4) + 4(-2)\right]$$

$$= \left[-\tfrac{32}{3} + 8 + 16\right] - \left[\tfrac{4}{3} + 2 - 8\right]$$

$$= -\tfrac{32}{3} + 24 - \tfrac{4}{3} + 6 = -\tfrac{36}{3} + 30 = -12 + 30 = 18$$