Section 6.2 – Volumes (Discs)

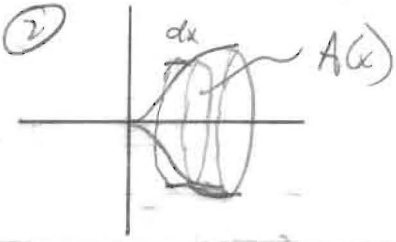
HW (463) 2, 5, 7, 9)

Top Board

① Rotation About y-axis $\Rightarrow$ $\int_a^b A(y)\,dy$

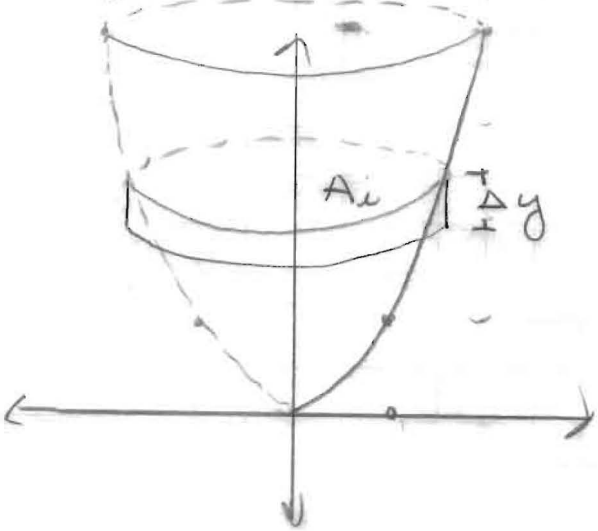
② Rotation About x-axis $\Rightarrow$ $\int_a^b A(x)\,dx$

① 

② 

(I) Rotation About Y-Axis ($x=0$)

i.e. $y = x^2 \qquad 0 \le x \le 2$



$A_i = \pi r_i^2 = \pi x_i^2$

$V_i = \pi x_i^2 \Delta y$

↳ can we express x_i^2 in terms of y_i?

$\Rightarrow y = x^2 \Rightarrow x_i^2 = y_i$

$\Rightarrow V_i = \pi y_i \Delta y$

$\Rightarrow$ goA ($n = 5, 10, 25$)

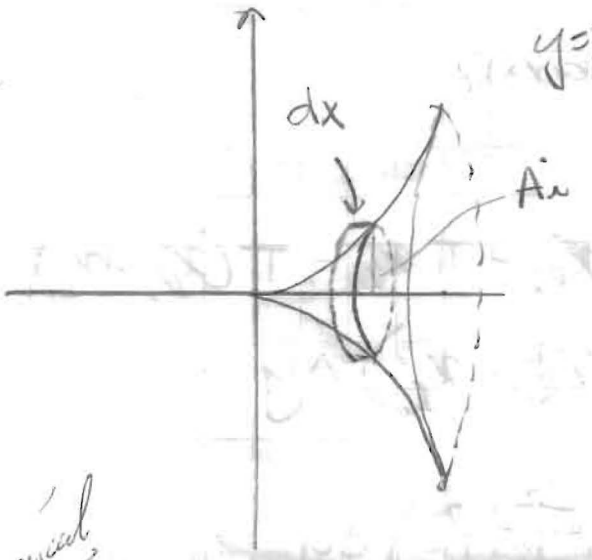
↑ numerical

$$V_{TOT} = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \pi y_i \Delta y$$

$$\Rightarrow \lim_{n\to\infty} V_{TOT} = \int_a^b \pi y\,dy \Rightarrow \text{for } x=0,\ y=0;\ x=2,\ y=4$$

$$\therefore \pi \int_0^4 y\,dy = \pi \tfrac{1}{2} y^2 \Big|_0^4 = \tfrac{\pi}{2}(16-0) = 8\pi$$

(II) ROTATION ABOUT X-AXIS ($y=0$)



$y=x^3$, $0\le x\le 1$

$A_i = \pi r_i^2 = \pi y_i^2$

$V_i = \pi y_i^2\, dx$

Since $y = x^3$

$\Rightarrow V_i = \pi x_i^6 dx$

Numerical

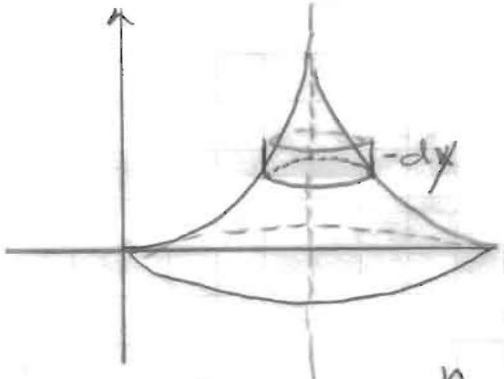
go BIG ($n=5, 10, 25$)

$$V_{TOT} = \sum_{i=1}^{n} \pi x_i^6\, dx \Rightarrow \lim_{n\to\infty} V_{TOT} = \int_0^1 \pi x^6\, dx$$

$$= \pi\int_0^1 x^6\, dx = \frac{\pi}{7}x^7\Big|_0^1 = \pi/7 \approx .1429\pi$$

(III) Rotation about $x=1$

$y=x^2$, $0\le x\le 1$



$A_i = \pi(x_i-1)^2$

$V_i = A_i\, dy = \pi(x_i-1)^2 dy$

$\Rightarrow y = x^2 \Rightarrow x = \sqrt{y}, \Rightarrow x_i = \sqrt{y_i}$

$\Rightarrow V_i = \pi(\sqrt{y_i}-1)^2 dy$

GoC $n=5, 10, 25$ — Numerical

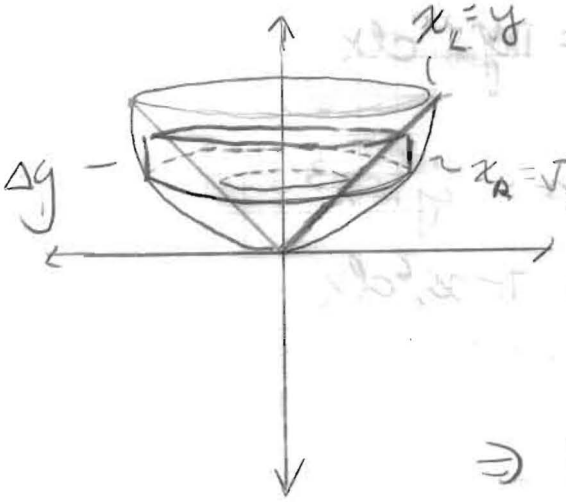
$$V_{TOT} = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \pi(\sqrt{y_i}-1)^2 dy$$

$$\lim_{N\to\infty} V_{TOT} = \pi\int_0^1 (\sqrt{y_i}-1)^2 dy = \pi\int_0^1 (y-2y^{1/2}+1)dy$$

$\Leftrightarrow y=0$ to $y=1$ $= \pi\left(\frac{1}{2}y^2 - \frac{4}{3}y^{3/2} + y\right) = \frac{1}{6}\pi$

(IV) Inner & Outer Function

$\left.\begin{matrix} y=x \\ y=x^2 \end{matrix}\right\} 0 \le x \le 1$ about y-axis



$$A_i = \pi x_{R_i}^2 - \pi x_{L_i}^2 = \pi (x_{R_i} - x_{L_i})^2$$

$$V_i = \pi (x_{R_i}^2 - x_{L_i}^2)\,\Delta y$$

$$V_i = \pi (\sqrt{y_i}^2 - y_i^2)\,\Delta y$$

$$\Rightarrow V_{TOT} = \sum_{i=1}^{n} V_i = \pi \sum_{i=1}^{n} \pi (y_i - y_i^2)\,\Delta y$$

$$\lim_{n\to\infty} V_{TOT} = \pi \int_0^1 (y - y^2)\,dy = \pi \left(\frac{1}{2}y^2 - \frac{1}{3}y^3\right)\Big|_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{6}\pi$$