Section 7.1 – Integration by Parts

DAY 1, 11/12

Set up MATLAB

HW:

TOP BOARD

Int by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int F'(x)g(x)dx$$

Let $u = f(x)$ $\quad dv = g'(x)dx$

$du = F'(x)dx$ $\quad v = g(x)$

$$\Rightarrow \boxed{\int u\,dv = uv - \int v\,du}$$

Use when you see products of functions.

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I Derivation

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\Rightarrow \int d[f(x)g(x)] = \int [f'(x)g(x) + f(x)g'(x)]dx$$

$$f(x)g(x) = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

u {
- I - inverse trig
- L - logarithmic
- A - any polynomial
- T - trig
- E - exponential

Example

$\int x \sin x \, dx$ Note: Substitution does not work

$\Rightarrow u = x \qquad v = -\cos x$

$du = dx \qquad \Rightarrow dv = \sin x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$$\Rightarrow \int x \sin x \, dx = -x\cos(x) - \int -\cos(x)\,dx$$

$$= -x\cos(x) + \int \cos(x)\,dx$$

$$= -x\cos(x) + \sin(x) + C$$

Example - Oops

$\Rightarrow u = \sin x \qquad v = \frac{1}{2}x^2$

$du = \cos x \, dx \qquad \Rightarrow dv = x\,dx$

$$\int u\,dv = uv - \int v\,du \Rightarrow \int x \sin x \, dx = \frac{1}{2}x^2 \sin x - \int \frac{1}{2}x^2 \cos x \, dx$$

don't go there!!

Example

$\int_0^1 x^2 e^x \, dx$

Parts Rule For Definite integral

$$\int_a^b f(x) g'(x) dx = f(x) g(x) \Big|_a^b - \int_a^b f'(x) g(x) dx$$

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

$\Rightarrow u = x^2$ $\quad v = e^x$

$du = 2x\,dx$ $\quad \Rightarrow dv = e^x dx$

$$\int_0^1 x^2 e^x dx = x^2 e^x \Big|_0^1 - \int_0^1 2x e^x dx$$

$$= (1)(e) - (0)(1) - 2\int_0^1 x e^x dx$$

$$= e - 2\int_0^1 x e^x dx$$

$\Rightarrow u = x$ $\quad v = e^x$

$du = dx$ $\quad \Rightarrow dv = e^x dx$

$$\int_0^1 x e^x dx = x e^x \Big|_0^1 - \int_0^1 e^x dx$$

$$= (1)(e) - (0)(1) - e^x \Big|_0^1$$

$$= e - 0 - (e - 1)$$

$$= 1$$

$\therefore \int_0^1 x^2 e^x dx = e - 2(1) = e - 2$

"go!"

Example

$$\int e^x \sin x \, dx$$

→ $u = \sin x$ $\quad v = e^x$

$du = \cos x \, dx$ $\quad \rightarrow dv = e^x dx$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$u = \cos(x)$ $\quad v = e^x$

$du = -\sin(x) dx$ $\quad dv = e^x dx$

$$\int e^x \cos(x) \, dx = e^x \cos x - \int e^x (-\sin x) \, dx$$

$$= e^x \cos x + \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin(x) - e^x \cos x - \int e^x \sin x \, dx$$

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin(x) - \cos(x))$$