7.7 – Approximate Integration   Set Up MATLAB

TOP BOARD

Midpoint Rule	
Trapezoid Rule	
Simpsons Rule	

I. Some Integrals Can't Be Done Analytically

i.e. $\int_0^1 e^{x^2}\,dx$ or $\int_{-1}^{1} \sqrt{1+x^3}\,dx$

– so we do them numerically

⇒ "GO!" (for $n = 4, 8, 16, 32$)

⇒ Preserve Data on SIDE BOARD

$\int_{-1}^{1}\sqrt{1+x^3}\,dx$

$n = 4$, $\Delta x = .5$

$M_4 = 1.9784$

$T_4 =$

$S_4 =$

How Do we get these?

II) MIDPOINT RULE

$$\int_a^b f(x)dx \approx M_n = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \dots f(\bar{x}_n)\right]$$

n = # intervals

$\Delta x = (b-a)/n$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

Example

$n=4$

$\Delta x = (2)/4 = .5$

i	x_{i-1}	x_i	$\bar{x}_i$	$f(\bar{x}_i)$	del x × f(x)
1	-1	.5	-.75	1.1924	.5962
2	-.5	0	-.25	1.0078	.5039
3	0	.5	.25	.9922	.4961
4	.5	1	.75	.7603	.3802
				3.9527	1.9764

III TRAPAZOID Rule

$$\int_a^b f(x) \approx T_4 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) \dots + 2f(x_{n-1}) + f(x_n)\right]$$

$n=4$

$\Delta x/2 = .5/2 = .25$

TRAP/SIMPSON

i	x_i	$f(x_i)$	m	
0	-1	0	1/1	0 / 0
1	-.5	.7071	2/4	1.4142 / 2.8284
2	0	1	2/2	2 / 4
3	+.5	1.2247	2/4	2.4495 / 4.8990
4	1	1.4142	1/1	1.4142 1.4142

$7.2779 \times \frac{\Delta x}{2} = 1.8195$

$11.14162 \times \frac{\Delta x}{3} = 1.8569$

IV Simpsons Rule

⇒ View Graph Again - Fits Parabolas to Even Number of Points

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

→ Build Off Previous Chart ← In Red

(V) Continue GO1 conclusions:

① n gets bigger - methods converge

② Approximation improves

V HW Prob 24/427

GO2