Section 7.8 - IMPROPER INTEGRALS

TOP BOARD

Type I	Type II
a) <u>If</u> $\int_a^t f(x)dx$ exists for all $t \geq a$, <u>then</u> $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$ b) If $\int_t^b f(x)dx$ exists for all $t \leq b$, then $\int_{-\infty}^b = \lim_{t\to-\infty} \int_t^b f(x)dx$ c) If both a) & b) true th $\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx$	a) If f is cont on [a,b) but discont at b, then $\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)dx$ if lim finite b) If f con (a,b] but discont at a then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$ is <u>finite</u>

Note: Type I Integral Goes to $\pm\infty$

Type II Discontinuity

I) <u>Consider:</u> $\Rightarrow$ Type I Set Up

Terms

$\int_1^\infty \frac{1}{x^{1/2}}dx = 2x^{1/2}\Big|_1^\infty = \infty - 2 = \infty$ divergent

$\int_1^\infty \frac{1}{x}dx = \ln x\Big|_1^\infty = \infty - 0 = \infty$ divergent

$\int_1^\infty \frac{1}{x^2} = -\frac{1}{x}\Big|_1^\infty = -\frac{1}{\infty} - \left(-\frac{1}{1}\right) = 0 + 1 = 1$

- convergent
- Improper Int. Type I

II) <u>Definition</u> - Top Board - Type I

III Consider: Type II: Set Up

all are discont at 0

$$\int_0^1 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_0^1 = -1-\left(-\frac{1}{0}\right) = -1+\frac{1}{0} = -1+\infty = \infty \quad \text{divergent}$$

$$\int_0^1 \frac{1}{x}\,dx = \ln x\Big|_0^1 = 0-\ln 0 = 0-(-\infty) = \infty \quad \text{divergent}$$

$$\int_0^1 \frac{1}{x^{1/2}}\,dx = 2x^{1/2}\Big|_0^1 = 2-0 = 2 \quad \to \text{convergent}$$

→ Imp Int Type II

IV Example

$$\int_2^\infty \frac{1}{(x+3)^{3/2}}\,dx \Rightarrow \text{Type I}$$

$$\Rightarrow \frac{-2}{(x+3)^{1/2}}\Big|_2^\infty = \frac{-2}{\infty} - \frac{-2}{(5)^{1/2}} = \frac{2}{\sqrt{5}}$$

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