

# SECTION 8.3 - Center of Mass

(485) 27, 29, 30, 33

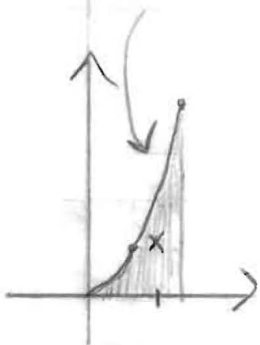
TOP BOARD - NO DERIVATION

Centroid (under a curve)	Centroid (Between Curves)
$\bar{x} = \frac{1}{A} \int_a^b x f(x) dx$	$\bar{x} = \frac{1}{A} \int_a^b x [f(x) - g(x)] dx$
$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)^2] dx$	$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)^2 - g(x)^2] dx$
(A = Area)	

## I. Example

$$y = x^2 \quad 0 \leq x \leq 2$$

where is centroid:



① Find A

$$\int_0^2 x^2 dx \rightarrow \frac{1}{3} x^3 \Big|_0^2 = \frac{8}{3}$$

$$\textcircled{2} \bar{x} = \frac{3}{8} \int_0^2 (x)(x^2) dx = \frac{3}{8} \int_0^2 x^3 dx$$

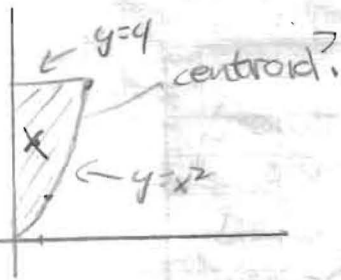
$$= \frac{3}{32} x^4 \Big|_0^2 = \frac{3}{32} (16) = \frac{3}{2} = \bar{x}$$

$$\textcircled{3} \bar{y} = \left(\frac{3}{8}\right) \left(\frac{1}{2}\right) \int_0^2 (x^2)^2 dx = \frac{3}{16} \int_0^2 x^4 dx$$

$$= \frac{3}{80} x^5 \Big|_0^2 = \frac{(3)(32)}{(80)} = \frac{(3)(2)}{5} = \frac{6}{5}$$

### EXAMPLE (Between Curves)

$$A = \int_0^2 (4-x^2) dx = 4x - \frac{1}{3}x^3 \Big|_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$$



$$\bar{x} = \frac{3}{16} \int_0^2 [x(4-x^2)] dx = \frac{3}{16} \int_0^2 (4x-x^3) dx$$

$$= \frac{3}{16} \left[ 2x^2 - \frac{1}{4}x^4 \right]_0^2 = \frac{3}{16} [8-4] = \frac{3}{4}$$

$$\bar{y} = \left(\frac{3}{16}\right) \left(\frac{1}{2}\right) \int_0^2 (4^2-x^4) dx = \frac{3}{32} \int_0^2 (16-x^4) dx$$

$$= \frac{3}{32} \left( 16x - \frac{1}{5}x^5 \right) \Big|_0^2 = \frac{3}{32} \left[ 32 - \frac{1}{5}(32) \right]$$

$$= \frac{3}{32} \left( \frac{4}{5} \right) (32) = \frac{12}{5}$$

### EXAMPLE MAPLE

$$y=x^2, y=x^3 \Rightarrow x \rightarrow 0 \dots 1$$