Chapter 8.3 - Hydrostatic Pressure

TOP BOARD

Physics	Hydro-Static Pressure
$P = F/A$	
metric ⇒ $\frac{\text{Newtons}}{\text{meter}^2}$ = Pascals	ρ = density
imperial ⇒ lbs/in^2 ⇒ psi	g = gravity
	d = depth
	$P = \rho g d$

I Example (metric)

water has density 1000 kg/m^3

gravity is 9.8 m/sec^2

What is pressure at 10 meters

$$P = \rho g d = (1)(9.8)(10) = 98\ N/m^2$$

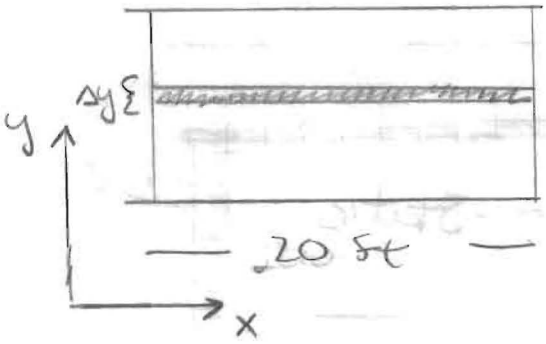
II Example (Imperial)

water has density 62.5 lbs/ft^3 ← note this is already pd

Pressure at 30 ft?

$$P = (\rho g) d = (62.5\ lbs)/ft^3\ 10\ ft\ \frac{625\ lbs}{ft^2}\left(\frac{ft^2}{144\ in^2}\right) = 4.34\ psi$$

(III) Example



Force on Rectangular Dam

Since, $P = F/A \Rightarrow F = PA = (\rho g)(dV/A)$

$\Rightarrow F_i = 62.5 \frac{lbs}{ft^3} (10 - y_i) ft \; (20) \Delta y \; ft^2$

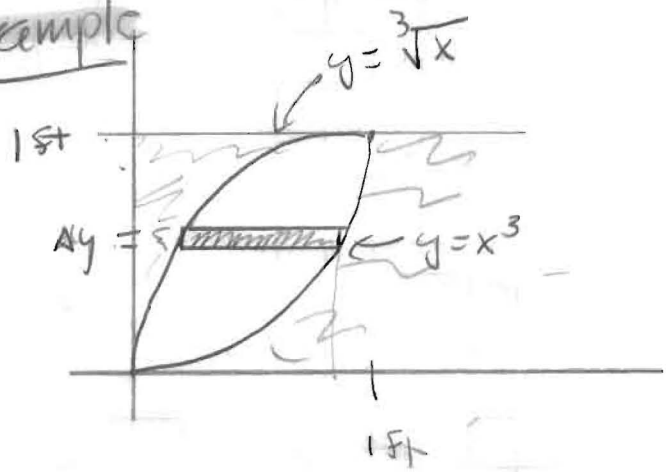
$= 1250 (10 - y_i) \Delta y \quad lbs$

$\Rightarrow$ if n increments $\Rightarrow \quad F = \sum_{i=1}^{n} 1250 (10 - y_i) \Delta y$

$\Rightarrow$ if $n \to \infty \qquad F = \int_0^{10} 1250 (10 - y) \, dy$

$= 1250 \left[10y - \frac{1}{2} y^2 \right]_0^{10} = 1250 (100 - 50) = \underline{62{,}500 \; lbs}$

(IV) Example



What is force on object?

$$F_i = P_i A_i \quad \Rightarrow A_i = \left(\sqrt[3]{y_i} - y_i^3\right)\Delta y$$

$$\Rightarrow P_i = (62.5)(1 - y_i)$$

$$\Rightarrow \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} (62.5)(1-y_i)\left(\sqrt[3]{y_i} - y_i^3\right)\Delta y$$

$$\Rightarrow n \to \infty \Rightarrow F = 62.5 \int_0^1 (1-y)\left(y^{1/3} - y^3\right) dy$$

$$\Rightarrow F = 62.5 \int_0^1 \left(y^{1/3} - y^3 - y^{4/3} + y^4\right) dy$$

$$= 62.5 \left[\tfrac{3}{4} y^{4/3} - \tfrac{1}{4} y^4 - \tfrac{3}{7} y^{4/3} + \tfrac{1}{5} y^5\right]_0^1$$

$$= 62.5\left[\tfrac{3}{4} - \tfrac{1}{4} - \tfrac{3}{7} + \tfrac{1}{5}\right] = 62.5\left[\tfrac{1}{2} - \tfrac{3}{7} + \tfrac{1}{5}\right]$$

$$= 62.5\left[\tfrac{35 - 10 + 14}{70}\right] = \left(\tfrac{29}{70}\right)62.5 \approx \underline{25.89 \text{ ft-lbs}}$$

(V) Example

Prob 19 (HW)