Section 9.1 Modeling w/ DE's - A Survey

I. Population Growth Model

$$\frac{dP}{dt} = kP \quad \text{where} \quad \begin{cases} k = \text{growth rate} \\ P = \text{number in population} \\ t = \text{time} \end{cases}$$

(a) $\Rightarrow$ Possible Solution... $P = e^{kt}$

$$\Rightarrow \frac{dP}{dt} = ke^{kt} = kP$$

(b) $\Rightarrow$ Problem with model ... its explosive

II. Another Model For Population Growth

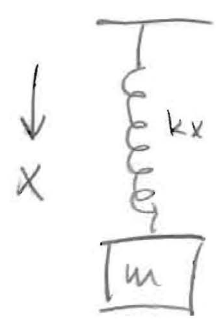
let K be the maximum population

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$$

analysis = $\begin{cases} P < K \Rightarrow \frac{dP}{dt} > 0 \\ P = K \Rightarrow \frac{dP}{dt} = 0 \\ P > K \Rightarrow \frac{dP}{dt} < 0 \end{cases}$

$\Rightarrow$ called logistics diff-eq, proposed by Dutchman Verhulst in 1840

(III) Spring-Mass System Motion



$F = -kx$ (hooks law)

$F = ma = m\frac{d^2x}{dt^2}$

$\therefore m\frac{d^2x}{dt^2} = -kx \Rightarrow$ possible solution:

$\Rightarrow$ possible solution: $x = \sin\sqrt{\frac{k}{m}}\,t$

(IV) General Diff-Eqs.

contain: $\begin{cases} \text{- unkown function} \\ \text{- one or derivaties} \end{cases}$

i.e. $y' = xy$ (recall $y' = \frac{dy}{dx}$)

$$y''' + 3y'' + y' = x^2y + \sin(xy)$$

- "order" of Diff-Eq. "Highest Derivative"

V Solving Diff-Eqs

$y' = 3t^5 \Rightarrow \frac{dy}{dt} = 3t^5 \Rightarrow \int dy = \int 3t^5 dt$

$\Rightarrow y(t) = \underline{\frac{1}{2}t^6 + C}$

← general solution - family of curves

⇒ "initial condition"... suppose $y(0) = 10$

$\Rightarrow y(0) = \frac{1}{2}(0)^6 + C = 10 \Rightarrow C = 10$

$\therefore \underline{y(t) = \frac{1}{2}t^6 + 10}$

← specific solution for initial cond

VII Examples

Prob 4, p. 511

Prob 5, p. 511

Prob 6, p. 511