Section 9.2 - Direction Fields

#33/34

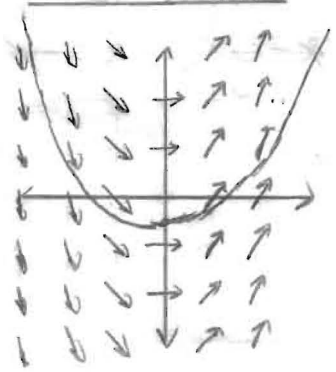
Top Board

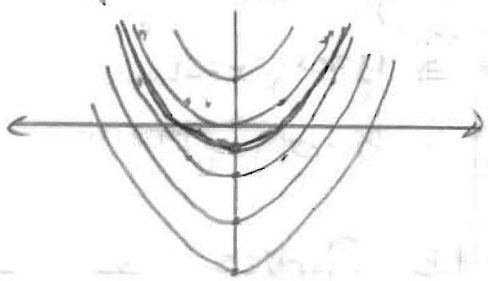
Diff-Eq's What Do they Mean

i.e. $\frac{dy}{dx} = x$ GOO

Direction Field or Family of Curves

$\Rightarrow y = \frac{1}{2}x^2 + C$





(I) Diff-Eq Tells me something about behaviour but does not give a unique solution

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but what IS in addition to the DEQ I give you info about a single point (i.e. a condition)

i.e. $y(3) = 4$ or when $x=3$, $y=4$

$\Rightarrow y = \frac{1}{2}x^2 + C \Rightarrow 4 = \frac{1}{2}(3)^2 + C \Rightarrow 4 = 4.5 + C \Rightarrow C = -.5$

$\therefore y = \frac{1}{2}x^2 - .5$ ← this is/is one curve

$\Rightarrow$ relate to flow field

(II) Example - Go!

$\frac{dy}{dx} = x+y$ } Draw a flow field

$\Rightarrow$ pick points $\Rightarrow$ $(0,-1)$ for straight line trajectory

(III) Doing the Same Thing in MAPLE $\Rightarrow$ Go to a

↳ Doing It on the Calc!!

(IV) Problem 5 (guessing a shape) - p. 520

$y' = y^2 - x^2$

$\Rightarrow y' = 0$ if $x^2 = y^2 \Rightarrow x = y$, $x = -y$

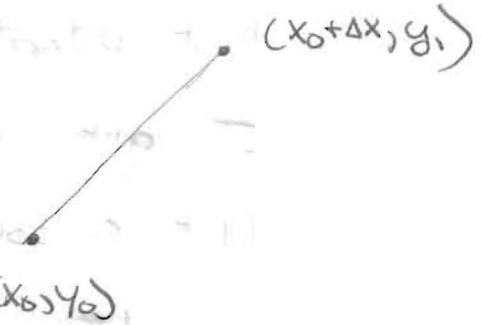
$\Rightarrow y = 0$, $x = \pm 1$ $\quad y' = -1$

$\Rightarrow x = 0$, $y = \pm 1$ $\quad \Rightarrow y' = 1$

$\Rightarrow$ No Curves so go2

(V) Eulers Method - How Do We Approximate The curve mathematically

Example! $y' = x+y$



① pick an increment Δx
② pick an initial point x_0, y_0
③ estimate y_1

$y' = \frac{y_1 - y_0}{\Delta x} \Rightarrow$ $\boxed{y_1 = y_0 + y'\Delta x}$

⇒ let $\Delta x = .2$
$(x_0, y_0) = (0, 1)$

⇒ what is y when $x = 2$

⇒ make a chart

$\Delta X = .2$

x_0	y_0	y'	y_1
0	1	1	1.2
.2	1.2	1.4	1.48
.4	1.48	1.88	1.86
.6	1.86	2.46	2.35
.8	2.35	3.18	2.98
1	2.98		

II Doing This on Excell - Demo

x y dy/dx * Δx Δy