9.3 Separable Equation - Day 1 #36

TOP BOARD

If I can seperate the functions FOR A 1^{ST}-ORDER DE I CAN SOLVE The System

i.e. or i.e

$$\frac{dy}{dx} = g(x)f(y) \qquad \frac{dy}{dx} = \frac{g(x)}{h(y)}$$

$$\Rightarrow \frac{dy}{f(y)} = g(x)dx \qquad \Rightarrow h(y)dy = g(x)dx$$

Example

$$\frac{dy}{dx} = \frac{6x^2}{2y+\cos y} \Rightarrow (2y+\cos y)dy = 6x^2 dx$$

now integrate both sides:

$$\int 2y+\cos y \, dy = \int 6x^2 \, dx$$

$$\Rightarrow \boxed{y^2+\sin y = 2x^3+C}$$

How Plot? $\Rightarrow$ $x = \left\{\frac{1}{2}\left[y^2+\sin y - C\right]\right\}^{1/3}$

① USE PARAMATRIC PLOTTER ON CALC

$\Rightarrow$ let $C=0 \Rightarrow$
$y=t$, $x = \left\{\frac{1}{2}\left[t^2+\sin t\right]\right\}^{1/3}$

Example

$$\frac{dy}{dx} = x^2 y \Rightarrow \int \frac{dy}{y} = \int x^2 dx$$

$$\ln y = \tfrac{1}{3}x^3 + c \Rightarrow \underline{\underline{\ln y - \tfrac{1}{3}x^3 = c}}$$

Demo Plots on MAPLE Example

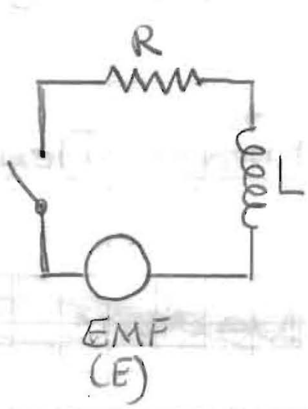
$\boxed{\text{? Turkey PROPLEM - Day 2 ?}}$

9.3. – Seperable Equations – Day 2 – Examples

Example 1 → RL – Circuit

① ⇒ Recall DEQ for RLC-CKT

$$L\frac{dI}{dt} + RI + \cancel{\frac{1}{C}Q} = E$$



② ⇒ No Capacitor, Assume R, L, C are Constant

$$\Rightarrow L\frac{dI}{dt} + RI = E \Rightarrow L\frac{dI}{dt} = E - RI \Rightarrow \boxed{\frac{dI}{dt} = \frac{E-RI}{L}}$$

↑ Flow Fields Based on this

$$\Rightarrow L\,dI = (E - RI)\,dt$$

$$\Rightarrow L\int \frac{dI}{E-RI} = \int dt$$

initial condition for current

$$\Rightarrow -\frac{L}{R}\ln|E-RI| = t + C$$

$$\Rightarrow \ln|E-RI| = -\frac{R}{L}(t+\ )$$

$$\Rightarrow E - RI = e^{-R/L\,(t+C_0)} = e^{-R/L\,t}\overbrace{e^{-R/L\,C}}^{\text{constant } K} = ke^{-R/L\,t}$$

$$\Rightarrow E - ke^{-R/L\,t} = RI$$

$$\Rightarrow I = \frac{E}{R} - \underbrace{\left(\frac{k}{R}\right)}_{\text{still a constant} = K} e^{-R/L\,t} = \frac{E}{R} - Ke^{-R/L\,t}$$

③ Initial Condition: $I(0) = I_0$

$$\Rightarrow I(0) = \frac{E}{R} - k = I_0 \Rightarrow k = \frac{E}{R} - I_0$$

$$\therefore \boxed{I(t) = \frac{E}{R} - \left(\frac{E}{R} - I_0\right)e^{-R/L\,t}}$$

(II) Let $R = 50\,\Omega$, $L = 25\,h$, $E(t) = 100$, $I(0) = 0$

$$\Rightarrow I(t) = \frac{100}{50} - \left(\frac{100}{50} - 0\right)e^{-\frac{50}{25}t} = \underline{\underline{2 - 2e^{-2t}}}$$

<u>Maple Demo</u>

(III) <u>Newtons Law of Cooling Example</u>

$$\frac{dT}{dt} = k(T - T_0) \qquad \begin{cases} T = \text{temperature} \\ k = \text{cooling constant} \\ T_0 = \text{Room Temperature} \end{cases}$$

$$\int \frac{dT}{T - T_0} = \int k\,dt \Rightarrow \ln|T - T_0| = kt + C$$

$$\Rightarrow T - T_0 = e^{kt+c} = e^{kt}e^{c} = A e^{kt}$$

$$\Rightarrow T = T_0 + A e^{kt} \Rightarrow \text{Assume } T(0) = T_i \text{ (initial temp)}$$

$$\Rightarrow T(0) = T_0 + A = T_i = A = T_i - T_0$$

$$\Rightarrow \boxed{T(t) = T_0 + (T_i - T_0)e^{kt}}$$

<u>Turkey Problem</u>

At noon $(t=0)$ $T_{\text{TURKEY}} = 400°$

Room Temp $= 75°$

At 1 PM $(t=1)$ $T_{\text{TURKEY}} = 300°$

$$T(t) = 75 + (325e^{kt})$$

$$T(1) = 75 + 325e^{k} = 300 \Rightarrow e^{k} = \frac{225}{325} = \frac{9}{13} \Rightarrow k \simeq -.3677$$

When is Turkey $150° \Rightarrow 75 + 325e^{-.3677t} = 150$

$\Rightarrow 3.98$ hrs $\approx$ 4:00 PM