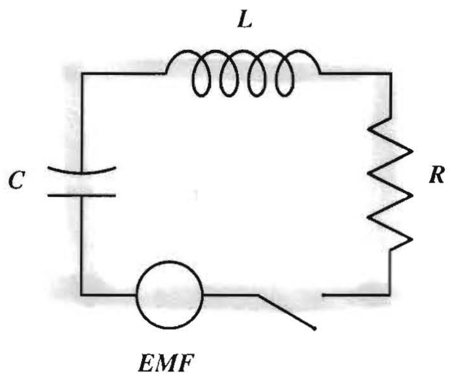
Introduction: Modeling Electric Circuits

One of the topics that you will study in your Physics and Electrical Engineering courses at the Naval Academy is that of electric circuits. Consider the single-loop series circuit below.



[a] The device labeled EMF, which stands for *electromotive force*, can be a battery or a generator. The EMF supplies current to the circuit.

[b] The device labeled C is a *capacitor* which stores charge.

[c] The device labeled R is a *resistor* which opposes the flow of current.

[d] The device labeled L is an *inductor* which opposes change in the flow of the current.

The last device in the circuit is a switch, shown open. When the switch is closed, a current $I(t)$ runs through the circuit which changes the charge $Q(t)$ at the capacitor. The following table summarizes what the measurements are, their symbols, and units.

Circuit Measurements		
measurement	**symbol**	**units**
time	t	seconds (sec)
charge	$Q(t)$	Coulombs
current	$I(t)$	Amperes (amps)
electromotive force (EMF)	$E(t)$	Volts (V)

When the current is flowing, there is a corresponding voltage drop across each of the devices. The next table summarizes the device symbols, units, and voltage drops.

Circuit Devices			
device name	**symbol**	**units**	**voltage drop**
capacitor	C	Farads (F)	$E_C = \frac{1}{C} \cdot Q$
resistor	R	Ohms (Ω)	$E_R = R \cdot I$
inductor	L	Henries (H)	$E_L = L \cdot \frac{dI}{dt}$

At any time t, the current $I(t)$ and the charge $Q(t)$ are related by two important rules:

1. The current equals the rate of change of the charge with respect to time (1 amp = 1 coulomb/sec):

$$I(t) = \frac{dQ(t)}{dt}$$

2. **Kirchhoff's Law:** (the electrical equivalent of Newton's second law of motion)

The sum of the voltage drops around the closed circuit is equal to the voltage supplied: $E_L + E_R + E_C = E$

or

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = E$$

Example 1

Since $I = \frac{dQ}{dt}$, we get

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E$$

Example 4

which is a second order ordinary differential equation — or **ODE** — for the charge $Q(t)$.

There are two special cases that we can solve with elementary methods.

R-C Circuits: If there is no inductor, we can investigate a first order ODE for the charge Q

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E$$

R-L Circuits: If there is no capacitor, we can investigate a first order ODE for the current I

$$L\frac{dI}{dt} + RI = E$$

A *solution* to an ODE is any function that satisfies the ODE. If we know the initial state of the circuit — namely the charge or current measurements — then we have an *initial condition* or **IC**. An ODE plus an IC is called an *initial value problem* or **IVP**.