Chapter 7 & 8
Team B

Integration by parts

$\int u\,dv = uv - \int v\,du$

Tabular method x^2e^x

u (take der.)		v (take int.)
x^2	+	e^x
$2x$	−	e^x
2	+	e^x
0		e^x

$= x^2e^x - 2xe^x + 2e^x + C$

"U" hierarchy
Logarithmic function
Inverse trig function
Any polynomial
Trig function
Exponential function

Partial fractions

(I) $\dfrac{1}{(x\pm a)(x\pm b)} = \dfrac{A}{(x\pm a)} + \dfrac{B}{(x\pm b)}$

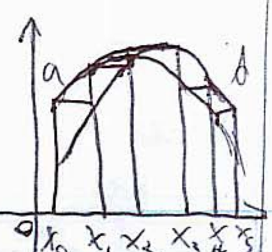
(II) $\dfrac{1}{(x\pm a)^2(x\pm b)} = \dfrac{A}{(x\pm a)^2} + \dfrac{B}{(x\pm a)} + \dfrac{C}{(x\pm b)}$

*if not factorable ↓

(III) $\dfrac{1}{(x^2\pm a)(x\pm b)} = \dfrac{Ax+B}{(x^2\pm a)} + \dfrac{C}{(x\pm b)}$

7.7

Approx. Integration

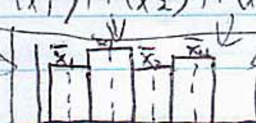


- Some areas can be approximately determined
- $(b-a)/n = \Delta x$..
- You can divide $[a,b]$ into n intervals of equal length

Left endpoint approx	right endpoint approx.
$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_{i-1})\Delta x$	$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i)\Delta x$

Midpoint rule → take f of middle value between the two ends of the subinterval

$\int_a^b f(x)\,dx \approx M_n \approx \Delta x[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_n)]$



Trapezoidal Rule

$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]$

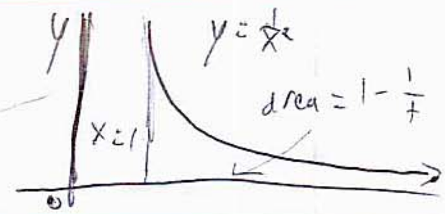
Simpson's Rule

$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+\ldots+f(x_n)]$

*rotate 2 and 4 except 1st and last

7.8

Improper Integrals



If f has an infinite discontinuity, then it is an imp. integral.

→ $A(t) = \int_1^t \frac{1}{x^2}\,dx = -\frac{1}{x}\Big]_1^t = 1 - \frac{1}{t}$

a) If $\int_a^t f(x)\,dx$ exists for every # $t \geq a$, then $\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$

b) If $\int_t^b f(x)\,dx$ exists for every # $t \leq b$, then $\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$, provided this limit exists (as a finite #)

* Convergent → if lim exists. Divergent → if lim. DNE

c) If both $\int_a^\infty f(x)\,dx$ and $\int_{-\infty}^b f(x)\,dx$ are convergent, then $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$

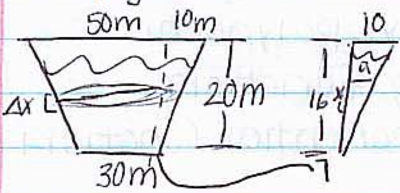
* ... This is used to find areas of curves that are different than normal.

Hydrostatic Forces - Find the force of water on an object or surface.

$F = pAd = mg$ F = force, p = density, A = area, d = depth, m = mass

g = gravity.

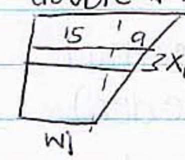
$P = pgd - \delta d$



$\frac{10}{20} = \frac{9}{16-x_i}$

$\frac{1}{2} = \frac{16-x_i}{2}$

double it for other side



$w_i = (15+a)2$

$w_i = 30+2a$

$= 30+16-x_i$

$46-x_i$

depth of water is 16m

$P = 1000\,g\,x_i$ $g = 9.8$

limits are depths of water

Area $= w_i \Delta x = (46-x_i)\Delta x$

$F = PA = 1000 g x_i (46-x_i)\Delta x$

$$\int_0^{16} 1000g(46x - x^2)\,dx$$

$$= 9.8 \cdot 1000 \int (46x - x^2)\,dx$$

$$= 9800\left[23x^2 - \frac{x^3}{3}\right]_0^{16}$$

$$\boxed{4.43 \times 10^7 \text{ N}}$$

Center of Mass:

Center of mass (Under curve): $\bar{x} = \frac{1}{\text{AREA}} \int_a^b x f(x)\,dx$

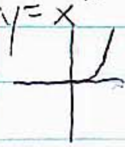
$\bar{y} = \frac{1}{\text{AREA}} \int_a^b \frac{1}{2}[f(x)^2]\,dx$

Centeroid (between curves): $\bar{x} = \frac{1}{\text{AREA}} \int_a^b x[f(x) - g(x)]\,dx$

$\bar{y} = \frac{1}{\text{AREA}} \int_a^b \frac{1}{2}[f(x)^2 - g(x)^2]\,dx$

Example Under Curve:

$y = x^2$ $0 \le x \le 2$



Area $= \int^b x^2\,dx = \int_0^2 x^2\,dx = \frac{8}{3}$

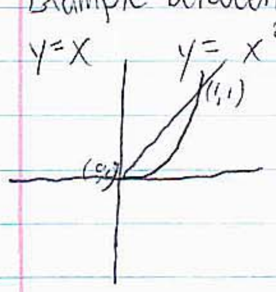
$\bar{x} = \frac{3}{8} \int_0^2 x(x^2)\,dx = \frac{3}{8}\int_0^2 x^3\,dx = \frac{3}{8}\cdot\left[\frac{x^4}{4}\right]_0^2 = \frac{3}{2}$

$\bar{y} = \left(\frac{3}{8}\right)\left(\frac{1}{2}\right)\int_0^2 (x^2)^2\,dx = \frac{3}{16}\int_0^2 x^4\,dx = \frac{3}{16}\cdot\left[\frac{x^5}{5}\right]_0^2 = \frac{6}{5}$

Center of Mass $\bar{x}, \bar{y}$ $\left(\frac{3}{2}, \frac{6}{5}\right)$

Example Between Curves:

$y = x$ $y = x^2$



Area $= \int_0^1 (x - x^2)\,dx = \frac{x^2}{2} - \frac{x^3}{3} = \frac{1}{6}$

$\bar{x} = 6\int_0^1 x(x - x^2)\,dx = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$

$\bar{y} = 6\int_0^1 \frac{1}{2}[(x)^2 - (x^2)^2]\,dx = 6\int_0^1 \frac{1}{2}[x^2 - x^4]\,dx = 3\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{2}{5}$

$(\bar{x}, \bar{y}) = \left(\frac{1}{2}, \frac{2}{5}\right)$