Team D

11.1: Sequence

$\{a_1, a_2, a_3\}$ or $\{a_n\}_{n-1}^{\infty}$

ex. $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ $a_n = \frac{n}{n+1}$: $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4} \dots\right\}$

* Convergent : LIMIT EXISTS

* Divergent : DNE

11.2: Series

- Theorem #1: Divergence

If $\lim_{n\to\infty} a_n = \begin{cases} \neq 0 \\ \text{DNE} \end{cases}$, then $S = \sum_{n=1}^{\infty} a_n$ diverges

If $\lim_{n\to\infty} a_n = 0 \rightarrow$ converges

$a_n = \frac{1}{n}$, $\lim_{n\to\infty} \frac{1}{n} = 0$ however, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

- Theorem #2: Disatisfying

If $S = \sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

- Theorem #3: Geometric

$S = \sum_{n=1}^{\infty} a_r^n$ where $-1 < r < 1$ if 1) series converges
2) $S = \frac{a}{1-r}$

Sequence : Does it Converge?

Series : What does it converge to? How close?

$$S = \sum_{n=1}^{\infty} a_n \rightarrow \text{① geometric} : ar^{n-1}$$

$$S = \sum_{n=1}^{\infty} a_n \rightarrow \text{② alternating} : (-1)^n b_n$$

$$S = \sum_{n=1}^{\infty} a_n \rightarrow \text{③ P-series} : \frac{1}{n^p}$$

Power Series : For what values of "X" does the power series converge?

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

- Maclaurin Series

$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$ SET UP CHART!

ex. $f(x) = \frac{1}{1-x}$

n	$f^n(x)$	$f^n(0)$	$f^{(n)}(0)/n!$
0	$1/1-x$	1	1
1	$1/(1-x)^2$	1	1
2	$-2/(1-x)^3$	2	1
3	$6/(1-x)^4$	6	1

$\frac{1}{1-x} = \sum_{n=0}^{\infty} (1) x^n = \sum_{n=0}^{\infty} x^n$

- Taylor Series

$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n$

ex. $f(x) = x^2 - 2x + 3 \rightarrow$ center on 3

n	$f^{(n)}(x)$	$f^{(n)}(a)^3$	$\frac{f^{(n)}(a)}{n!}$
0	x^2-2x+3	6	6
1	$2x-2$	4	4
2	2	2	1
3	cannot keep going		

answer: $2(x-3)^2 + 4(x-3) + 6$

Team D

Power Series

converges $-1 < x < 1$

diverges $|x| \geq 1$

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 \ldots$$

Power Series centered at a

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 \ldots$$

Theorm for a given series $\sum_{n=0}^{\infty} c_n (x-a)^n$ [illegible] 3 possib[illegible]

1) series converges only when $x = a$

2) series converges for all x

3) positive number R such that the series converges if $|x-a| < R$ diverges if $|x-a| > R$

Example find the radius of convergence and interval of convergence [illegible]

$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ ← let be equal to a_n

$$\left| \frac{a_{n+1}}{a_n} \times \text{reciprocal of } a_n \right| = \frac{(-3)^{n+1} x^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(-3)^n x^n} = -3x\sqrt{\frac{n+1}{n+2}} \text{ as } n \to \infty$$

$3x$ $\quad -1 < 3x < 1 \quad$ $\underbrace{-\frac{1}{3} < x < \frac{1}{3}}_{\text{interval of convergence}}$ Radius

Power series as function

theorm

if power series $\sum c_n (x-a)^n$ has a radius of convergence $R > 0$ the function f is differentiated by

$F(x) = c_0 + c_1(x-a) + c_2(x-a)^2 \ldots = \sum_{n=0}^{\infty} c_n (x-a)^n$ is differentiable and continuous on interval $(a-R, a+R)$

1 $f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 \ldots = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$

2 $\int f(x)\,dx = C + c_0(x-a) + c_1\left(\frac{(x-a)^2}{2}\right) \ldots = C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$

Ex find power series of $\frac{x^3}{x+2}$

$$\frac{x^3}{x+2} = x^3 \cdot \frac{1}{x+2} = x^3 \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n+3} = \frac{1}{2}x^3 - \frac{1}{4}x^4 + \frac{1}{8}x^5 - \frac{1}{16}x^6 \ldots$$