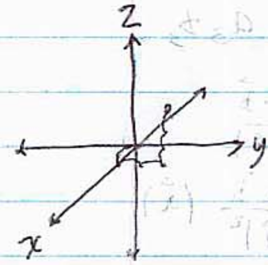
CHAPTER 12 - VECTORS

12.1 THREE-DIMENSIONAL COORDINATES



(x, y, z)
$(1, 2, 3)$

xy PLANE ; $z=0$
xz PLANE, $y=0$
yz PLANE, $x=0$

$i = x$ axis
$j = y$ axis
$k = z$ axis

DISTANCE FORMULA

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

EQUATIONS OF SPHERE

RADIUS $= r$
CENTER $= (a,b,c)$
$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

12.2 VECTORS - INDICATES A QUANTITY THAT HAS BOTH MAGNITUDE & DIRECTION

LENGTH OF 2-D VECTOR (MAGNITUDE)

$\vec{a}=(a_1, a_2)$ $|a|=\sqrt{a_1^2+a_2^2}$

LENGTH OF 3-D VECTOR (MAGNITUDE)

$\vec{a}=(a_1, a_2, a_3)$ $|a|=\sqrt{a_1^2+a_2^2+a_3^2}$

ADDING VECTORS $\vec{a}+\vec{b}=\langle a_1+b_1, a_2+b_2, a_3+b_3\rangle$

SUBTRACTING VECTORS $\vec{a}-\vec{b}=\langle a_1-b_1, a_2-b_2, a_3-b_3\rangle$

VECTOR PROPERTIES

1. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
2. $\vec{a}+(\vec{b}+\vec{z})=(\vec{a}+\vec{b})+\vec{z}$
3. $\vec{a}+0=\vec{a}$
4. $\vec{a}+(-\vec{a})=0$
5. $\vec{c}(\vec{a}+\vec{b})=\vec{c}\vec{a}\ \vec{c}\vec{b}$
6. $(\vec{c}+\vec{b})\vec{a}=\vec{c}\vec{a}+\vec{d}\vec{a}$

POINTS TO A VECTOR

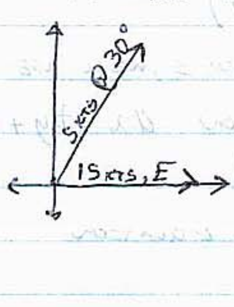
$A=(1,3,6)$ $B=(-2,7,2)$

$\vec{AB}=((-2-1),(7-3),(2-6))$

UNIT VECTOR - LENGTH OF ONE

$\Rightarrow \vec{u}=\dfrac{\vec{a}}{|\vec{a}|}$ (DIVIDE THE VECTOR BY ITS MAGNITUDE)

EXAMPLE OF CRAFT PROBLEM



1. FIND x, y VALUES FOR EACH VECTOR'S END POINT
$(5\cos(60), 5\sin(60))$ $(15\cos(0), 15\sin(0))$
2. ADD THEM TOGETHER AND USE PATHAGOREAN THEOREM TO FIND NEW VECTOR
$\sqrt{(17.5)^2+(2.5\sqrt{3})^2}=18.03$ KTS
3. USE INVERSE TAN TO GET ANGLE
$\tan^{-1}\left(\dfrac{2.5\sqrt{3}}{17.5}\right)=14.92°$ $90°-14.92\approx 75°$

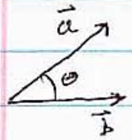
12.3 THE DOT PRODUCT

$\vec{a} \pm \vec{b} = \vec{c}$, $\vec{c}$ IS A VECTOR

$\vec{a} \cdot \vec{b} = n$, n IS A SCALAR (NUMBER)

IF $\vec{a} \cdot \vec{b} = 0$, $\theta = 90°$

IF $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$, THEN $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$



$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\text{comp}_a b = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

$\text{proj}_a b = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} (\vec{a})$

CROSS PRODUCT

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

$\vec{a} \times \vec{b} = \vec{c}$, so $\vec{c} \perp \vec{a}$ + $\vec{c} \perp \vec{b}$

$$\begin{array}{cc} & \begin{array}{ccc} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \end{array} \\ \vec{a} \times \vec{b} = & \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{array}$$

$$+i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1 \rangle$$

12.5 EQUATIONS OF LINES AND PLANES

$P = (1, 0, -2)$ $\vec{PQ} = \langle 0, 1, 3 \rangle$

VECTOR FORM: $\langle x, y, z \rangle = \langle 1, 0, -2 \rangle + \langle 0, 1, 3 \rangle t$

↑ INITIAL POINT (P)

↑ VECTOR MADE BY 2 POINTS IN THIS CASE $\vec{PQ}$.

PARAMETRIC FORM:

(x y + z values evaluated with t, using the equation above)

$x = 1$

$y = t$ or $\langle x, y, z \rangle = \langle 1, t, -2+3t \rangle$

$z = -2 + 3t$

EQUATION OF PLANE FROM 3 POINTS, P, Q, + R.

1. FIND NORMAL LINE
   ($\vec{PQ} \times \vec{PR} = \langle 10, -3, 1 \rangle$)
2. SELECT ONE OF THE POINTS, WE'LL USE $P = (1, 0, -2)$
3. PLUG BOTH INTO EQUATION $ax + by + cz = d$ to find d
   ($10(1) + -3(0) + 1(-2) = 8$)
4. USE d TO WRITE FINAL EQUATION
   $10x - 3y + z = 8$

DISTANCE BETWEEN POINT + PLANE

$$= \frac{|ax_1 + cy + dz + d|}{\sqrt{a^2 + b^2 + c^2}}$$