I. 5.3: Fundamental Theorem of Calculus

part 1: If f is continuous on $[a,b]$, then the function g defined by $g(x) = \int_a^x f(t)dt$ is continuous on $[a,b]$ and differentiable on (a,b), and $g'(x) = f(x)$.

part 2: If f is continuous on $[a,b]$, $\int_a^b f(x)dx = F(b) - f(a)$, where F is a function such that $F'(x) = f(x)$.

II. 5.5 Substitution

- Substitution Rule: if u=g(x) is differentiable AND has a range that is an interval AND "f" is continuous on the interval, **THEN** $\int f(g(x)) \times g'(x)dx = \int f(u)du$

Indefinite: Ex. 1) Find $\int \sin(5x)\,dx$

let $u = 5x$

$\frac{du}{dx} = 5$

$du = 5dx$

$dx = \frac{1}{5}du$

$\frac{1}{5}\int \sin(u)\,du$

$-\frac{1}{5}\cos(u) + C$

$\boxed{-\frac{1}{5}\cos(5x) + C}$

Definite Integral: if "g" is continuous on [a,b] and "f" is continuous on the range u=g(x), then: $\int f(g(x)) \cdot g'(x)dx = \int f(u)du$

*to evaluate definite integrals using Substitution, must change the Limits of Integration

Definite: Ex. 2)

$\int_0^1 x^2(1+2x^3)^5\,dx$

let $u = 1+2x^3$

$du = 6x^2\,dx$

$\frac{1}{6}du = x^2dx$

$x=0 \quad x \Rightarrow u = 1 + 2(0)^3 = \boxed{1}$

$x=1 \quad x \Rightarrow u = 1 + 2(1)^3 = \boxed{3}$

$\int_1^3 \frac{1}{6}u^5du$

$\frac{1}{6}\cdot\frac{1}{6}u^6\Big|_1^3$

$\frac{1}{36}u^6\Big|_1^3 = \frac{1}{36}[3^6 - 1^6] = \boxed{\frac{182}{9}}$

III. 6.1 Areas Between Curves

The area bounded by two functions $f(x)$ and $g(x)$ between $x = a$ and $x = b$, given that f and g are continuous on $[a,b]$ and $f(x) > g(x)$ for all x in $[a,b]$, is:

$$\int_a^b [f(x) - g(x)]\,dx$$

Example: Find the area of the region bounded above by $y=e^x$, bounded below by $y=x$, and bounded on the sides by $x=0$ and $x=1$

$A = \int_0^1 (e^x - x)\,dx = e - 1.5$

IV. 6.2 Volumes

Solids of revolution

--determine orientation of cylinders (dx or dy "slices")

--determine radius of disks

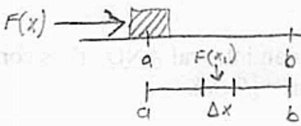
--use formula $V = \pi r^2 h$, where $h = \Delta x$ or Δy

--integrate

V. 6.4 Work

- Work= Force x Distance=(F)(D)
- Force= Mass x Gravity constant
  - Gravity constant= 9.8 m/s^2 OR 32ft/s^2
- Hooke's Law: $F_{spring} = KX$

Work with variable Force: Ex. 1)

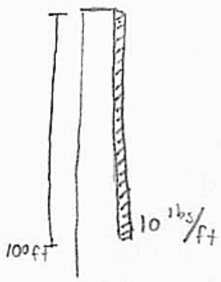


$$W_i = F(x_i)\Delta x$$

$$\sum_{i=1}^{n} W_i = \sum_{i=1}^{n} F(x_i)\Delta x$$

when "n" $\to \infty$: $w = \int_a^b F(x)\,dx$

Pulling up building: Ex. 2)



$W = F_i d_i$

$F_i = 10 \frac{lbs}{ft}\, \Delta x\, ft = 10\,\Delta x\, lbs$

$d_i = x_i$

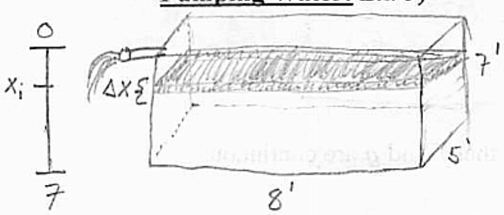
$W = 10x_i\,\Delta x$ ft-lbs

$W = \sum_{i=1}^{n} 10x_i \Delta x$

$$W = \int_0^{100} 10x\,dx$$

$$W = 5x^2\Big|_0^{100} = \boxed{50{,}000\ \text{ft lbs}}$$

Pumping Water: Ex. 3)



$V_{slice} = lwh = 8(5)(\Delta x)$

$V_{slice} = 40\,\Delta x\, ft^3$

$W_{slice} = Density_{H_2O}\,(V_{slice})$

$= 62.5 \frac{lbs}{ft^3}\,(40\,\Delta x\, ft^3) = 2500\,\Delta x$ lbs

$$W_T = \sum_{i=1}^{n} 2500x_i\Delta x$$

$$W_T = \int_0^7 2500x\,dx$$

$$W_T = 1250x^2\Big|_0^7$$

$$\boxed{W_T = 61250}$$

VI. 6.5 Average Value of a Function

Average Value of a Function:

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

Mean Value Theorem for Integrals:
If f is cont. on $[a, b]$, there is a number c in $[a, b]$ that:

$$f(c) = f_{ave}$$

or

$$\int_a^b f(x)dx = f(c)(b-a)$$

Example: Find the average value of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$

$$f_{ave} = \frac{1}{2+1}\int_{-1}^{2}(1+x^2)\,dx = 2$$