SUMMARY: CHAPTER 7&8

INTEGRATION BY PARTS: $\int u\,dv = uv - \int v\,du$

The order in which the value of u is determined:

L → logarithm
I → inverse trig. fcn
A → any polynomial fcn
T → trig. function
E → exponential fcn
D → dv

Partial Fractions:

example: $\int \frac{1}{x^2-1} = \int \frac{1}{(x+1)(x-1)} = \int \frac{A}{(x+1)} + \int \frac{B}{(x-1)}$

$$\longrightarrow \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \longleftarrow$$

APPROXIMATE INTEGRALS

Midpoint Rule

$\Delta x = \frac{b-a}{n}$

b = upper limit of integration
a = lower limit of integration
n = number of integrals.

$$\int_a^b f(x)\,dx = \Delta x\left(f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)\right)$$

$\hookrightarrow \bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) =$ midpoint of (x_{i-1}, x_i)

Trapezoid Rule

$\Delta x = \frac{b-a}{n}$

b = upper limit of integration
a = lower limit of integration
n = number of integrals

$x_i = a + i\Delta x$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_n)\right]$$

Simpson's Rule

$\Delta x = b-a/n$

b = upper limit of integration
a = lower limit of integration
n = number of integrals

← assume n is even

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

IMPROPER INTEGRALS:

$\int_a^{\infty} f(x)\,dx$ and $\int_{-\infty}^{b} f(x)\,dx$ → CONVERGENT if corresponding limit exists
→ DIVERGENT if corresponding limit does not exist

$\int_1^{\infty} \frac{1}{x^p}\,dx$ → convergent if $p > 1$
→ divergent if $p \leq 1$

$\int_a^b f(x)\,dx$ → convergent if corresponding limit exists.
→ divergent if corresponding limit does not exist.

Center of Mass:

In Summary:

$\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx$

$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx$

Region between 2 curves

$\bar{x} = \frac{1}{A}\int_a^b x[f(x)-g(x)]\,dx$

$\bar{y} = \frac{1}{A}\int_0^b \frac{1}{2}\{[f(x)]^2-[g(x)]^2\}\,dx$

$M_x = \sum_{i=1}^{n} m_i y_i$

n = number of "m" values

$m_i = m_1, m_2, m_3 \ldots$

y_i = corresponding y value of $P_1, P_2, P_3 \ldots$

n = "

m_i = "

$M_y = \sum_{i=1}^{n} m_i x_i$

x_i = corresponding x-value of $P_1, P_2, P_3 \ldots$

$m = \sum_{i=1}^{n} m_i \rightarrow$ add all of the "m" values

$\bar{x} = M_y/m \rightarrow$ x value of center of mass

$\bar{y} = M_x/m \rightarrow$ y value of center of mass

$\rightarrow$ results in a coordinate $\leftarrow$

Hydrostatic Force:

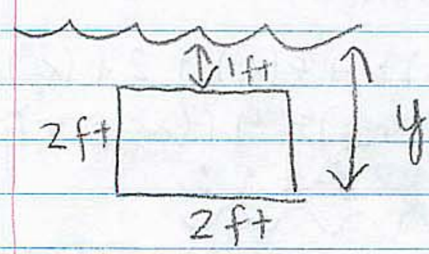
FORCE = PRESSURE × AREA

Area = yw dy → y = height of object
w = width

Pressure = Force × depth

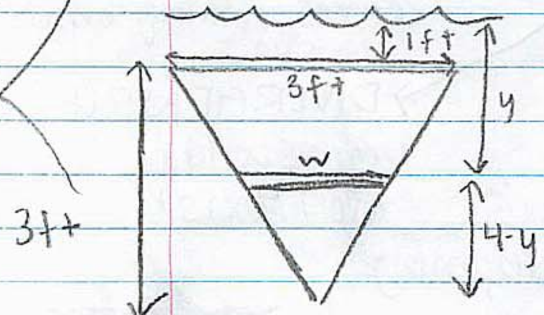
constant → density of water = 62.5 lbs/ft³

I.E.



$62.5\,ft^3 \int 2y\,dy \rightarrow 125\int_1^3 y\,dy \rightarrow 125\frac{y^2}{2}\Big|_1^3$

$125\left(\frac{9}{2}-\frac{1}{2}\right) \rightarrow 125\left(8/2\right) = 125 \cdot 4 =$ $\boxed{F = 500\text{ lbs}}$



$A = yw\,dy$

$62.5\int_1^4 yw\,dy \rightarrow 62.5\int_1^4 y(4-y)\,dy \rightarrow 62.5\int_1^4 4y-y^2\,dy$

$62.5\left(2y^2 - \frac{1}{3}y^3\right)\Big|_1^4 = 562.5\text{ lbs}$

$\frac{3}{3} = \frac{w}{4-y}$

$w = 4-y$