Calculus II Final

9-10 & Exponential Decay

Differential Equations

- Used to model-
  - population, radioactive decay, Electric circuit problems, mass spring systems.
- half life: $t_{1/2} = \frac{0.693}{k} = \frac{\ln 1/2}{k}$

population model 1: $\frac{dp}{dt} = kp$

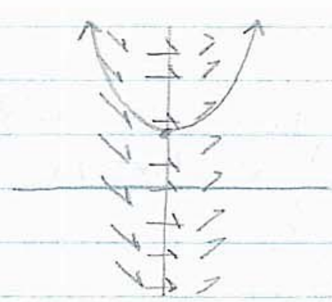
model 2: $\frac{dp}{dt} = kp\left(1 - \frac{p}{c}\right)$ * c - carry capacity
if $p = c$ then $\frac{dp}{dt} = 0$

Direction Fields

- $\frac{dy}{dx} = 2x$

* $\frac{dy}{dx}$ = differential eq because it has a derivative (slope of tangent line)
  2x - represents slope feild

- Initial condition → $y(0) = 1$
  ↳ when "x = 0" "y = 1"



"Slope feild" - as initial condition changes graph moves up and down

Eulers Method:

given: $\frac{dy}{dx} = 2x$, $y(0) = 1$ question: $y(.3) = ?$ stepsize: $\Delta x = .1$

$\Delta y = \frac{dy}{dx} \Delta x$

$X_{new} = X_{old} + \Delta X$

$Y_{new} = Y_{old} + \Delta Y$

step	x	y	$\frac{dy}{dx}$	$\cdot\ \Delta x$	$= \Delta y$
0	0	1	0	.1	0
1	.1	1	.2	.1	.02
2	.2	1.02	.4	.1	.04
3	.3	1.06			

$\Delta y = \frac{dy}{dx} \cdot \Delta x$

$y(.3) = 1.06$

Radioactive decay

- $\frac{dm}{dt} = km$ m = mass of radioactive obj
  t = time (sec/hrs/years)
  k = constant (k<0, negative)

- Population model

  $\frac{dP}{dt} = kP$ P = population $P(0) = P_0$
  t = time (min/hrs/yrs)
  k = constant (k>0 positive)

- $\frac{dM}{dt} = km \Rightarrow \int \frac{dm}{M} = \int k\,dt \Rightarrow \ln|M| = kt + c$

  $\Downarrow$

  $M = e^{kt+c} = M_0 e^{kt}$

  $M(t_{1/2}) = \frac{1}{2}M_0$

  $\hookrightarrow t_{1/2} = (\ln\frac{1}{2})(\frac{1}{k})$

Electrical Circuts

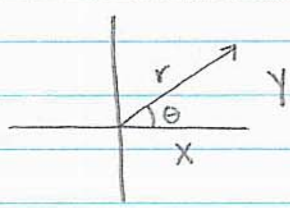
- Kirkoff's Law

voltage
$$E = iR + L\frac{di}{dt}$$
$$L\frac{di}{dt} + Ri = E$$

current
$$E = iR + \frac{1}{C}q$$
$$E = \frac{dq}{dt} + \frac{1}{C}q$$
$$R\frac{dq}{dt} + \frac{1}{C}q = E$$

* 'E' (volts)
'R' Resistance (ohms)
'L' inductor (henrys)
'C' capacitor (farads)
i - current (amps)
q - charge (coulombs)

Polar Coordinates



$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x} + \pi$ (if x < 0)

$r \to P$ (know (x, y)) $x^2 + y^2 = r^2 \Rightarrow r = \sqrt{x^2+y^2}$

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$P \to r$ $\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$A = \frac{1}{2}\int_a^b [f(\theta)]^2\,d\theta = \frac{1}{2}\int_a^b r^2\,d\theta$