11.1 Sequences

Definitions

- sequence = an infinite list of numbers
- generic sequence = $\{a_n\}^{\infty} = \{a_1, a_2, a_3, a_4, \text{etc.}\}$
- convergent = if a_n has a limit
- divergent = if the limit does not exist
- monotonic = a_n is either increasing or decreasing; not alternating
- bounded above or below = if $a_n \le M$ or $a_n \ge M$ for all n ("M" is any number)
- Factorial: $0! = 1$; $1! = 1$; $2! = 2$ $(1 \cdot 2)$; $3! = 6$ $(1 \cdot 2 \cdot 3)$; etc.

Theorems

1) if a_n is bounded and monotonic, then a_n is convergent
2) if $\lim_{n\to\infty} |a_n| = 0$, then the $\lim_{n\to\infty} a_n = 0$
3) if $a_n = r^n$ (r = any real #); $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < n \\ 1 & \text{if } r = 1 \\ \text{DNE} & \text{if } r \text{ is any other value} \end{cases}$
4) Squeeze Theorem: if $a_n \le b_n \le c_n$ for all $n \Rightarrow \lim_{n\to\infty} a_n \le \lim_{n\to\infty} b_n \le \lim_{n\to\infty} c_n$

Examples

1) $\left.\frac{n}{n+1}\right|_{n=0}^{\infty} = \{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \text{etc}\}$

Geometric Series

$a_n = ar^{n-1}$ where $-1 < r < 1$

ex) $S = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

11.2 Series

Definitions

- generic series: $S_n = a_1 + a_2 + \dots + a_n$ or $S = \sum_{n=1}^{\infty} a_n$

ex) $a_n = n$ $a_n = \{1, 2, 3, 4, \text{etc}\}$ $S_n = \{1, 3, 6, 10, 15, \dots, \frac{(n)(n+1)}{2}\}$

ex) (of geometric) $\sum_{n=1}^{\infty} 3\left(\frac{-1}{4}\right)^{n-1} = \frac{3}{1+\frac{1}{4}} = \frac{3}{5/4} = \frac{12}{5} = 1.2$

• Alternating Series: let $S = \sum_{n=1}^{\infty} (-1)^{n-1} b_n \Rightarrow$ if $\lim_{n\to\infty} b_n = 0$ and $b_{n+1} < b_n$, it converges

ex) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right)$ $\lim_{n\to\infty} b_n = 0$ and $b_{n+1} < b_n$ $\left(\frac{1}{2} < \frac{1}{1}\right)$, so the series converges

· P Series: $S = \sum_{n=1}^{\infty} \frac{1}{n^p}$ if $0 \le p \le 1 \rightarrow$ Diverges

$p > 1 \rightarrow$ Converges

· The Power Series:

$\sum_{n=0}^{\infty} C_n x^{n} = C_0 + C_1 x + C_2 x^2 + C_3 x^3$ any $f(x) = \sum_{n=0} C_n x^n$ "any function can be expressed as a power series"

Ex: $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 \ldots = \frac{1}{1-x}$, $-1 < x < 1$

Ratio Test: $S = \sum_{n=1}^{\infty} a_n \rightarrow$ if $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \begin{cases} < 1 \text{ convergent} \\ > 1 \text{ divergent} \\ = 1 \text{ no conclusion} \end{cases}$

· Maclaurin Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

· Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$