

Chapter 12

3D Coordinates

x, y, z



Distance between points

$$P_1 = (x_1, y_1, z_1)$$

$$P_2 = (x_2, y_2, z_2)$$

$$|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2 \quad \text{if at origin}$$

$$x^2 + y^2 + z^2 = r^2$$

Addition / Subtraction of vectors

$$\langle 3, 4, 5 \rangle + \langle 1, 3, 6 \rangle = \langle 4, 7, 11 \rangle$$

$$\langle 3, 4, 5 \rangle - \langle 1, 3, 6 \rangle = \langle 2, 1, -1 \rangle$$

Magnitude of vectors

$$|\vec{a}| = \langle a, b, c \rangle = \sqrt{a^2 + b^2 + c^2}$$

Dot products

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Theorem

$$a \cdot b = |a| |b| \cos \theta$$

$$\text{Comp}_a b = \frac{a \cdot b}{|a|}$$

$$\text{Proj}_a b = \frac{a \cdot b}{|a|^2} a$$

$$w = |F| |D| \cos \theta$$

Cross product

$$a \langle a_1, a_2, a_3 \rangle \quad b \langle b_1, b_2, b_3 \rangle$$

$$a \times b = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

or

$$\begin{array}{ccc} + & - & + \\ i & j & k \\ | & & | \end{array}$$

Theorem

$$|a \times b| = |a||b| \sin \theta$$

Torque

$$\tau = r \times F$$

Equations of lines and planes

$$\langle x, y, z \rangle = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

$$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$$

lines

Example

$$\langle 5, 1, 3 \rangle \text{ parallel to } \langle 1, 4, -2 \rangle$$

$$x = 5 + t \quad y = 1 + 4t \quad z = 3 - 2t$$

$$\langle a, b, c \rangle \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\text{or } ax + by + cz + d = 0$$

plane

Distance from a pt to a plane

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$