0755-1055 TUESDAY 12 DEC 2006

Calculators may be used in any capacity, but no other aids are allowed.

This exam consists of two parts, each worth 50%. The multiple choice problems should be answered on the bubble sheet. The long answer questions should be answered on paper or blue books. Follow any further instructions from your instructor.

PART ONE: Multiple choice (50%). The first 20 problems are multiple-choice. Fill in the best answer on the bubble sheet. **Write your name, alpha code, and section on your bubble sheet and bubble in your alpha code.** There is no penalty for wrong answers on the multiple-choice. Follow your instructor's instructions regarding scratch work.

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1. The equation $x^2 + 2x + (y-2)^2 + z^2 = 24$ represents

(A) a sphere with center (1, -2, 0) and radius 5.
(B) a circle with center (1, -2, 0) and radius $\sqrt{24}$.
(C) a sphere with center (-1, 2, 0) and radius $\sqrt{24}$.
(D) a sphere with center (2, 2, 0) and radius $\sqrt{24}$.
(E) a sphere with center (-1, 2, 0) and radius 5.

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2. Where does the line with vector equation $\mathbf{r}(t) = \langle t, 1+t, 2-3t\rangle$ meet the plane $x+y+z=4$?

(A) Where $t = 1$. (B) $(1, 0, 3)$ (C) $(-1, 0, 5)$ (D) $(1, 2, 1)$ (E) $(0, 1, 2)$

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3. Consider the three points A$(1, 0, 0)$, B$(2, 1, 1)$ and C$(3, 1, 0)$. A vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(A) $\langle 1, 2, 1\rangle$ (B) $\langle -1, 2, -1\rangle$ (C) $\langle -1, -2, -1\rangle$ (D) $\langle -1, -1, 2\rangle$ (E) $\langle 2, 1, 0\rangle$

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4. Parametric equations for a line through (0, 1, 0) and perpendicular to the plane $x + y + z = 1$ are

(A) $x = 0 - t,\ \ y = 1 - t,\ \ z = 0 - 2t$
(B) $x = 2 + 2t,\ \ y = 0 + t,\ \ z = -1 - t$
(C) $x = 1,\ \ y = 1 + t,\ \ z = 1$
(D) $x = 0,\ \ y = 1 - t,\ \ z = 0 + t$
(E) $x = 0 + 2t,\ \ y = 1 + 2t,\ \ z = 0 + 2t$

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5. It takes 4N of force to hold a spring stretched 2m past its natural length. The work required to stretch the spring from 2m past its natural length to 3m past its natural length is closest to

(A) $2J$ (B) $3J$ (C) $6J$ (D) $10J$ (E) $4N$

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6. The parabola $y = x^2$ can be parameterized by the equations $x = t$ and $y = t^2$. The length along the curve $y = x^2$ from (0, 0) to (2, 4) is closest to

(A) 3.5 (B) 5 (C) 6 (D) 7 (E) 12

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7. A car moves in a straight line for 30 seconds. Speeds are measured at 5 second intervals as shown in the table:

Time (sec)	0	5	10	15	20	25	30
Speed (m/s)	20	25	30	50	30	15	0

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The estimate of the distance traveled using Simpson's rule is closest to

(A) $700m$ (B) $750m$ (C) $800m$ (D) $830m$ (E) $860m$

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8. The power series expansion for $\sin(x)$ about $x = 0$ starts out

(A) $\sin(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ (B) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ (C) $\sin(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$
(D) $\sin(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ (E) $\sin(x) = 1 + x + x^2 + x^3 + \cdots$

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9. The area enclosed by 1 petal of the curve described by $r = sin(2\theta)$ is closest to

(A) 0 (B) 0.25 (C) 0.4 (D) 0.5 (E) 0.6

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10. The substitution $u = x^2$ transforms the integral $\int_0^4 \frac{x}{1+x^2}\,dx$ to

(A) $\int_0^2 \frac{2}{1+u}\,du$ (B) $\int_0^4 \frac{1}{2(1+u)}\,du$ (C) $\frac{1}{2}\int_1^{17} \frac{1}{u}\,du$ (D) $\frac{1}{2}\int_0^{16} \frac{1}{(1+u)}\,du$ (E) $\frac{1}{2}\int_0^{16} \frac{\sqrt{u}}{1+u}\,du$

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11. The radioactive isotope ${}^{238}U$ has a half-life of 4.5 billion years. A sample of ${}^{238}U$ found deep in the Earth's core has partially decayed into other elements and only 58% of the original ${}^{238}U$ remains. This data gives an estimate for the age of the Earth that is closest to

(A) 4 months (B) 3.3 billion years (C) 3.6 billion years
(D) 3.7 billion years (E) 13.7 billion years

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12. A polar form of the Cartesian equation $(x-1)^2 + y^2 = 1$ is

(A) $r = 2\cos(\theta)$ (B) $r = 2\sin(\theta)$ (C) $r = \cos(\theta)$ (D) $r = \sin(\theta)$ (E) $r^2 = 2\cos(\theta)$

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13. If y is a solution to the differential equation $y'(t) = y(t)\,(10 - y(t))$ and $y(0) = 1$, then a slope or direction field shows that as $t \to \infty$ $y(t)$ tends to

(A) -10 (B) 0 (C) 10 (D) ∞ (E) $y(t)$ has no limiting value.

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14. Which of the following integrals gives the area of the region bounded by the graphs of the equations $y = x^2$ and $y = x + 2$?

(A) $\int_0^4 (\sqrt{y}) - (y-2)\,dy$ (B) $\int_{-1}^2 (-x) - (-\sqrt{x+2})\,dx$ (C) $\int_{-2}^2 (x+2) - (x^2)\,dx$
(D) $\int_{-1}^2 (x^2) - (x+2)\,dx$ (E) $\int_{-1}^2 (x+2) - (x^2)\,dx$

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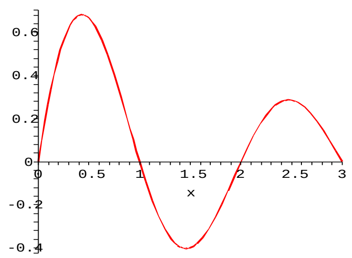
15. Suppose that **u**, **v** and **w** are vectors. Each of the following is a vector, a scalar, or makes no sense. Which makes no sense?
(A) $(\mathbf{u}\cdot\mathbf{v})\mathbf{w}$ (B) $(\mathbf{u}\times\mathbf{v})\times\mathbf{w}$ (C) $(\mathbf{u}\times\mathbf{w})\cdot(\mathbf{u}\times\mathbf{v})$ (D) $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$ (E) $(\mathbf{u}\cdot\mathbf{v})\cdot\mathbf{w}$

16. A trigonometric substitution transforms the integral $\int\sqrt{9+x^2}\,dx$ to
(A) $\int\sqrt{9+9\tan\theta}\,d\theta$ (B) $\int\sqrt{9+9sec^2(\theta)}\,d\theta$ (C) $\int\sqrt{9+9\tan^2(\theta)}\,d\theta$
(D) $3\int\sqrt{9+9\tan^2(\theta)}\sec^2(\theta)\,d\theta$ (E) $\int\sqrt{9+sin^2(\theta)}\cos^2(\theta)\,d\theta$

17. If $\int_1^\infty f(x)\,dx$ converges then $\int_1^t f(x)\,dx$ could be
(A) e^t-1 (B) $\frac{t^2+1}{t}$ (C) $\frac{e^{-t}}{t}$ (D) $\ln\left(\frac{1}{t}\right)$ (E) $t\sin(t)$

18. Given that $f(1)=3$ and $\int_0^1 f(x)\,dx=5$ then $\int_0^1 xf'(x)\,dx$ equals
(A) 3 (B) -3 (C) 0 (D) -2 (E) 2



19. Given the graph of $y=g(x)$ on the left, if we define $G(x)=\int_0^x g(t)\,dt$, then G is **decreasing** on the interval(s)
(A) $0<x<0.5$ and $1.5<x<2.5$
(B) $0<x<1$ and $2<x<3$
(C) $0<x<0.5$ and $1.5<x<2$
(D) $1<x<2$
(E) $0.5<x<1.5$ and $2.5<x<3$

20. To solve the first order linear differential equation $2\frac{dQ}{dt}+4020Q=12$ for $Q(t)$ by the integrating factor method, the integrating factor would be:
(A) $4020t$ (B) e^{-2010t} (C) e^{2010t} (D) e^{4020t} (E) e^{12t}

PART TWO: The following 10 questions are long answer questions. For full credit you must show all your work.

21. Evaluate the integral $\int x^2e^x\,dx$ **using integration by parts**. Clearly indicate your method and identify your choices for u and dv in order to receive full credit.

22. Find the volume of the solid obtained by revolving the region enclosed by the curves $y=x^2$, $y=0$ and $x=1$ about the y-axis using:
(a) the washer-disk method
(b) the shell method.

23. A Huey helicopter is hovering 1000 feet in the air directly above a 200lb midshipman. A cable weighing 1 lb per foot is used to raise the midshipman to the helicopter. Find the work done by the hoist.

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24. (a) Find the equation of the plane containing the three points $A(1, 0, 0)$, $B(2, 1, 1)$ and $C(3, 1, 0)$.
(b) Find the distance from the point $D(0, 0, 0)$ to your plane from part (a).

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25. A simple series electric circuit consists of a 0.05F capacitor, a 10Ω resistor and a battery supplying an electromotive force $E(t) = e^{-t}$. The initial charge on the capacitor is 0 Coulombs. Let $Q(t)$ be the charge on the capacitor after t seconds.
(a) Set-up an initial value problem for the unknown $Q(t)$.
(b) Solve the initial value problem **using the method of the integrating factor**.

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26. Find the area enclosed between the two curves $y = -x$ and $x = y^2 - 2$.

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27. Find the area of the part of the disk enclosed by $r = \cos(\theta)$ that lies outside the curve $r = 1/2$. [A picture of both curves might help earn part credit here.]

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28. Use Euler's method with 3 steps to estimate $y(6)$ if $y' = \frac{2}{1+x+y}$ and $y(0) = 0$.

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29. Willy Wonka's Chocolate Factory produces large amounts of chocolate milk. A large vat contains 100L of plain milk. A pipe feeds 10L/min of chocolate milk mixture containing 1kg of chocolate per liter of milk into the vat, which is kept well-mixed by several frothing fans. A uniform mixture drains from the vat at 10L/min. Let $Y(t)$ be amount (in kg) of chocolate in the tank at time t minutes.
(a) Find an initial value problem for $Y(t)$.
(b) Solve your IVP to find a formula for $Y(t)$.
(c) How long does it take until the mixture in the vat contains 50kg of chocolate?

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30. The Navy water tower near the football stadium consists of a spherical tank of radius 4m with its bottom 15m off the ground. How much work does it take to pump water from the ground to fill the tower? [Recall that the mass density of water is 1000kg/m^3 and use g = 9.8 N/kg.]