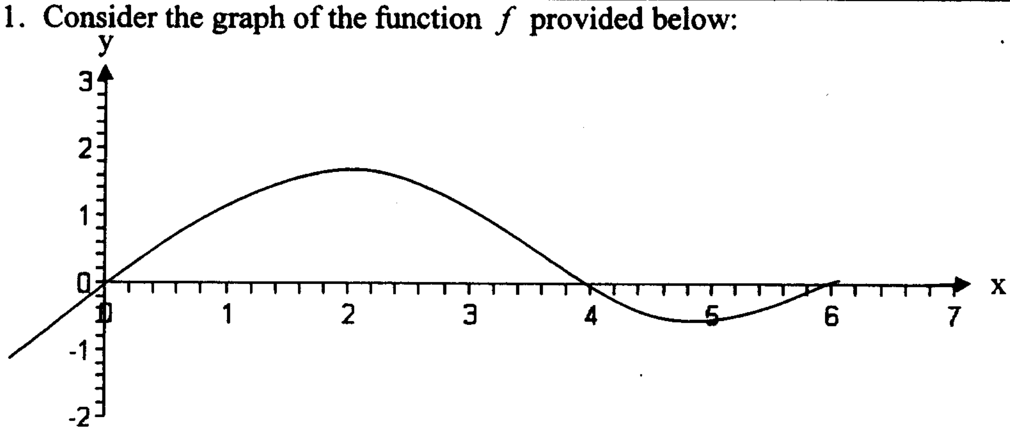
EXAM INSTRUCTIONS: There are 30 questions: 10 free answer questions (Exam Part I) and 20 multiple-choice questions (Exam Part II). Answer the questions of Part I on the bubble sheet provided and the questions of Part II in the blue books or on other paper provided by your instructor. DO NOT USE THIS QUESTION SHEET TO INDICATE YOUR ANSWERS.
Unless otherwise indicated, you may use your calculator to aid in answering any of the questions. You may not use any other reference material.

PART I: Multiple-Choice (50%)
Ensure your name, alpha code, and section number are filled in on the bubble sheet. Also ensure you fill in the bubbles associated with your alpha code. For each question of Part I, fill in the bubble associated with the letter which best answers the question. Your instructor may require your work to be shown for the multiple-choice questions in a blue book or on other paper in order to receive full or partial credit.

1. Consider the graph of the function f provided below:



Which of the following quantities is the largest?

a) $\int_0^6 f(x)dx$ b) $\int_4^6 f(x)dx$ c) $\int_0^4 f(x)dx$ d) $\int_0^2 f(x)dx$ e) $f'(2)$

2. The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = 2t - 2$, where $0 \le t \le 4$. The total distance traveled (distance over ground) by the particle is:

a) 6m b) 10m c) 16m d) 8m e) 12m

3. Assume we know the following about a function's values:

$x =$	1	3	5	7	9
$f(x) =$	2	3.7	4.1	1.3	.8

Using Simpson's Rule with n = 4, the approximate value for $\int_1^9 f(x)dx$ is closest to:

a) 10.33 b) 20.66 c) 21 d) 9.9 e) 19.8

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4. If $G(x) = \int_0^x \frac{1}{\sqrt{1-t^2}} dt$, then $G'(0) =$

a) $\frac{1}{\sqrt{1-x^2}}$ b) -1 c) 1 d) 0 e) arcsin(t)

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5. To solve for the integral $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$, the best substitution would be:

a) $u = \sqrt{x}$ b) $u = x$ c) $u = 1/\sqrt{x}$ d) $u = 1+\sqrt{x}$

e) cannot be solved using substitution

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6. $\int_0^2 \frac{1}{(x-1)^2} dx =$

a) $\left.\frac{-1}{(x-1)}\right|_0^2$ b) $\left.\frac{-2}{(x-1)^3}\right|_0^2$ c) $\left.\ln(x-1)^2\right|_0^2$

d) $\lim_{t\to\infty} \int_0^t \frac{1}{(x-1)^2} dx$ e) $\lim_{t\to 1^-} \int_0^t \frac{1}{(x-1)^2} dx + \lim_{t\to 1^+} \int_t^2 \frac{1}{(x-1)^2} dx$

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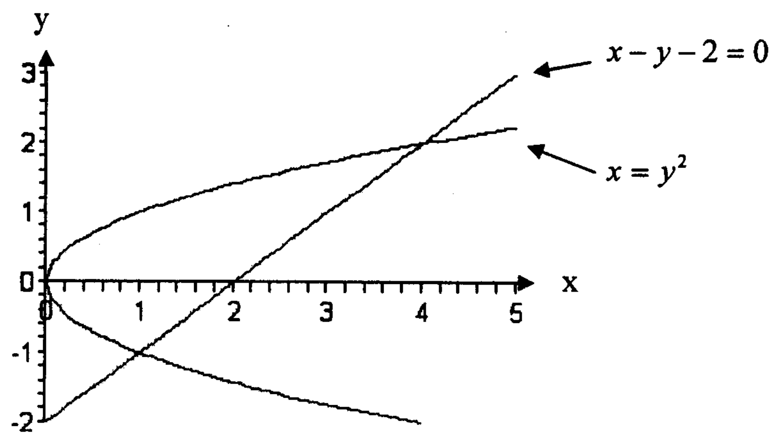
7. Which of the following integrals computes the area bounded by the curves $x = y^2$ and $x - y - 2 = 0$, pictured below:



a) $\int_0^4 (\sqrt{x} - x + 2)dx$ b) $\int_{-1}^2 (y^2 - y - 2)dy$ c) $\int_{-1}^2 (y + 2 - y^2)dy$

d) $\int_0^4 (\sqrt{x} - 2)dx$ e) $\int_1^4 (\sqrt{x} - x + 2)dx$

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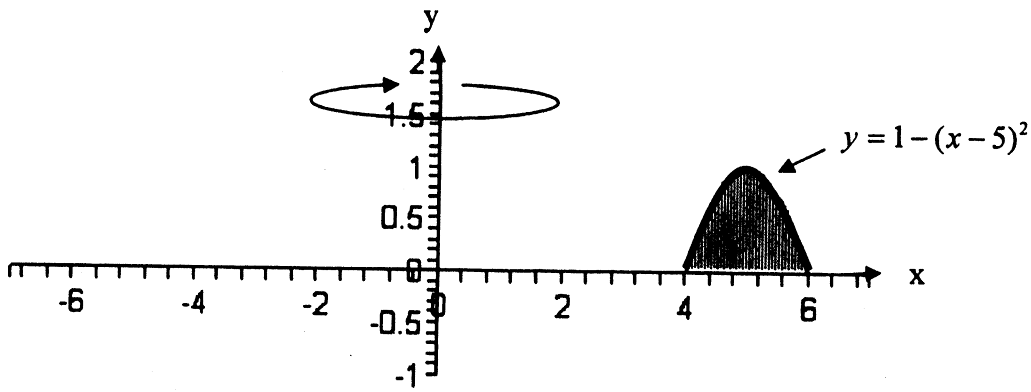
8. The region shown below is bounded by the curve $y = 1-(x-5)^2$ and the x-axis.



The integral to find the volume of the solid obtained by rotating this region around the y-axis is:

a) $\int_4^6 (1-(x-5)^2)dx$ b) $\int_4^6 2\pi x(1-(x-5)^2)dx$ c) $\int_4^6 \pi(1-(x-5)^2)^2 dx$

d) $\pi\int_0^1 (1-(x-5)^2)dx$ e) $\int_0^1 x(1-(x-5)^2)dx$

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9. Which of the following represents the arclength, L, of a parametric curve defined by $x = t^2$ and $y = t^3$ where $0 \le t \le 5$?

a) $L = \int_0^5 \sqrt{4t^2+9t^4}\,dt$ b) $L = \int_0^5 \sqrt{2t+3t^2}\,dt$ c) $L = \int_0^5 \sqrt{t^4+t^6}\,dt$

d) $L = \int_0^5 \sqrt{t^2+t^3}\,dt$ e) $L = \int_0^5 (4t^2+9t^4)dt$

10. An integral to find the area of the region enclosed by $r = 1+\sin\theta$ could be:

a) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\sin\theta)^2 d\theta$ b) $\int_0^{2\pi} (1+\sin\theta)^2 d\theta$ c) $\int_0^{\pi} (1+\sin\theta)^2 d\theta$

d) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+\sin\theta)^2}{2} d\theta$ e) $\int_0^{2\pi} (1+\sin\theta)d\theta$

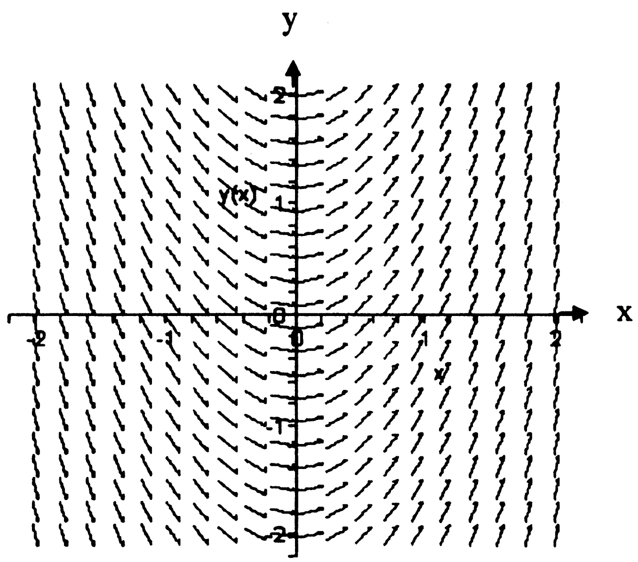
11. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work in foot-pounds is done in stretching the spring 3 feet beyond its natural position?

a) 900 b) 30 c) 150 d) 450 e) 300

12. For which value of r is e^{rt} a solution to the differential equation $\frac{dy}{dt} + 4y = 0$?

a) -4 b) 0 c) 4 d) 2 e) none of these

13. Which of the following differential equations has the direction field shown? (Direction fields are also known as slope fields)



a) $\frac{dy}{dx} = y$

b) $\frac{dy}{dx} = 2x$

c) $\frac{dy}{dx} = x + y$

d) $\frac{dy}{dx} = 1$

e) $\frac{dy}{dx} = x^2$

14. To solve the first order linear differential equation $\frac{dQ}{dt} + 4Q = 8$ for $Q(t)$ by the integrating factor method, the integrating factor would be:

a) $4t$ b) e^{-4t} c) e^{4t} d) e^4 e) $4t^2$

15. Which of the following first order differential equations models a simple electric circuit with a 20 Ohm resistor, a 4 Henry inductor, and a battery with a constant EMF = 100 Volts? (I = current, Q = charge, and t = time in seconds)

a) $5\frac{dQ}{dt} + Q = 25$ b) $\frac{dI}{dt} + 5I = 100$ c) $25\frac{dQ}{dt} + 4Q = 100$

d) $\frac{dI}{dt} + 5I = 25$ e) $20\frac{dI}{dt} + 4I = 100$

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16. What is the sum of the geometric series $1 + \frac{\pi}{4} + \frac{\pi^2}{16} + \frac{\pi^3}{64} + \cdots$?

a) $4 - \pi$ b) $\frac{4-\pi}{4}$ c) $\pi - 4$ d) 4π e) $\frac{4}{4-\pi}$

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17. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{3^n}$ is:

a) 1 b) 3 c) 5 d) 1/3 e) ∞

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18. Suppose the Taylor series for $f(x)$ centered at $a = 3$ is:

$$2 + 3(x-3) - \frac{5}{2}(x-3)^2 + \frac{1}{2}(x-3)^3 + \cdots$$

Then the graph of f at $x = 3$ is:

a) increasing and concave up

b) increasing and concave down

c) decreasing and concave up

d) decreasing and concave down

e) none of these

19. A vector of length 2008 in the opposite direction of the vector $7\vec{i} - 2\vec{j}$ is:

a) $2008(-7\vec{i} + 2\vec{j})$

b) $2008(7\vec{i} - 2\vec{j})$

c) $2008(\frac{7\vec{i}}{\sqrt{53}} - \frac{2\vec{j}}{\sqrt{53}})$

d) $2008(-\frac{7\vec{i}}{\sqrt{53}} + \frac{2\vec{j}}{\sqrt{53}})$

e) none of these

20. Which of the following is orthogonal to both $\vec{a} = \langle 3,-1,2 \rangle$ and $\vec{b} = \langle 2,2,-1 \rangle$?

a) $\langle 5,1,1 \rangle$

b) $\langle 1,-3,-3 \rangle$

c) $\langle -3,7,8 \rangle$

d) $\langle 6,-2,-2 \rangle$

e) $\langle 3,-7,8 \rangle$

PART II: Free Response (50%)

Solve the following 10 free response problems. Provide your answers in the blue books or on other paper provided by your instructor.

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21. Evaluate each of the following integrals without using your calculator. Show all work and intermediate steps needed to demonstrate the method of integration used. You may use your calculator to check your final answer.

a) $\int_0^4 \frac{x}{\sqrt{x^2+9}}dx$

b) $\int x\sin(x)dx$

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22. Find the average value of the animal population given by the formula $P(t) = 100 + 10t + (.02)t^2$ over the time interval $[0,10]$.

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23. Let R be the region bounded by the curves $y = x^2$ and the x-axis.

a) Sketch the region R.

b) Find $\bar{y}$, the y-coordinate of the centroid (center of mass) of the region.

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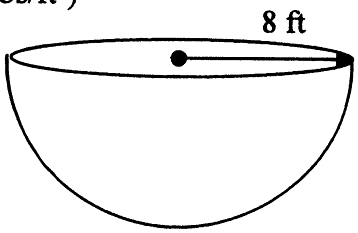
24. Assume the hemispherical tank with radius 8ft, as shown below, is full of water. Find the work required to pump all of the water out of the tank. (Water weighs 62.5 lbs/ft^3)



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25. Let R be the region that is outside of the curve $r = 2 + 2\cos\theta$ and inside of the curve $r = 2$.

a) Sketch the region R.

b) Find the area of this region.

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26. Consider the differential equation $\frac{dy}{dx} = y + 1$ with the initial condition $y(0) = 1$.

a) Apply Euler's method with a step size h = 1/4 to estimate the value $y(1/2)$.

b) Use the method of separation of variables to solve the differential equation and solve for the exact value of $y(1/2)$.

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27. The number of bacteria in a culture increases from 600 (at time t = 0) to 1800 in two hours. Assume that the rate of increase is directly proportional to the number of bacteria present at time, t. (i.e. $\frac{dy}{dt} = kt$ and $y(0) = 600$)

a) Solve the differential equation with the initial condition to find the formula for the number of bacteria for any time t.

b) At what time will the number of bacteria equal 5400?

28. A simple series circuit consists of a constant EMF of 50 Volts, a 10 Ohm resistor, and a .02 Farad capacitor.

a) Provide the first order differential equation which models this circuit.

b) Find a solution to the differential equation using the method of integrating factors. Assume that the charge on the capacitor is zero (0) when the switch is closed.

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29. Given the Maclaurin series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

a) Find the first four non-zero terms for the function $f(x) = e^{x^4}$.

b) Using only the first four non-zero terms found in part a) above, estimate the value of $\int_0^{0.5} e^{x^4}\,dx$.

30. Consider the vectors $\vec{a} = 4\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = -\vec{i} - 2\vec{j} + 2\vec{k}$

a) Find the dot product $\vec{a} \bullet \vec{b}$.

b) Find the angle between the two vectors $\vec{a}$ and $\vec{b}$.

c) Find the scalar projection of $\vec{b}$ onto $\vec{a}$ ($comp_a \vec{b}$).

d) Use the cross product $\vec{a} \times \vec{b}$ to find the area of the parallelogram determined by the vectors $\vec{a}$ and $\vec{b}$.