SM122 FINAL EXAM SOLUTIONS
SPRING 2005

① ⓒ $0 \le x \le 4$

② $v(t) = 0$ @ $t = 1$

$\left| \int_0^1 (2t-2)\,dt \right| + \left| \int_1^4 (2t-2)\,dt \right|$

$= \left| t^2 - 2t \Big]_0^1 \right| + \left| t^2 - 2t \Big]_1^4 \right| = |-1| + |9| = 10$ ⓑ

③ $\frac{2}{3}\left(2 + 4(3.7) + 2(4.1) + 4(1.3) + .8 \right) = 20.6667$ ⓑ

④ Let $F'(t) = \frac{1}{\sqrt{1-t^2}}$, then $\int_0^x \frac{1}{\sqrt{1-t^2}}\,dt = F(x) - F(0)$
$= G(x)$.

So $G'(x) = F'(x) - 0 = \frac{1}{\sqrt{1-x^2}}$; $G'(0) = \frac{1}{\sqrt{1-0^2}} = 1$ ⓒ

⑤ ⓐ Note $du = \frac{1}{2\sqrt{x}}\,dx$.

⑥ ⓔ

⑦ ⓒ

⑧ ⓑ

⑨ $x' = 2t$ $y' = 3t^2$ $\int_0^5 \sqrt{(x')^2 + (y')^2}\,dt = \int_0^5 \sqrt{4t^2 + 9t^4}\,dt$

ⓐ

⑩ None are correct; need $\int_0^{2\pi} \frac{1}{2}(1 + \sin\theta)^2\,d\theta$
or equivalent.

(11) $F = kx$; $100 = k\cdot(1)$; $k = 100$

$$\int_0^3 F\,dx = \int_0^3 100x\,dx = 50x^2\Big]_0^3 = 450 \text{ ft-lb.}$$ (d)

(12) $y = e^{rt}$ $y' = re^{rt}$ $y' + 4y = re^{rt} + 4e^{rt} = 0$

$\therefore r = -4$ (a)

(13) (b) Note $y' = 0$ when $x = 0$, $y' > 0$ when $x > 0$,
$y' < 0$ when $x < 0$.

(14) $e^{\int 4\,dt} = e^{4t}$ (c)

(15) $4\frac{dI}{dt} + 20I = 100$ (d)

(16) $\frac{1}{1-\frac{\pi}{4}} = \frac{4}{4-\pi}$ (e)

(17) Need $\lim\limits_{n\to\infty} \left| \frac{(n+1)^2(x-2)^{n+1}}{3^{n+1}} \right| \left| \frac{3^n}{n^2(x-2)^n} \right| < 1$

$$1 > \lim_{n\to\infty} \left|\frac{x-2}{3}\right| \left(\frac{n+1}{n}\right)^2 = \frac{|x-2|}{3}$$

So $|x-2| < 3$ (b)

(18) $f'(3) = 3 > 0$ $f''(3) = -5 < 0$ (b)

(19) $\| 7\vec{i} - 2\vec{j} \| = \sqrt{49+4} = \sqrt{53}$ $-\frac{2008}{\sqrt{53}}(7\vec{i} - 2\vec{j})$ (d)

(20) Check $\vec{v}\cdot\vec{a} = 0 = \vec{v}\cdot\vec{b}$: (c)

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(21) (a) $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$ Let $u = x^2+9$, $du = 2x\,dx$

$$\int_{x=0}^{x=4} \frac{x}{\sqrt{x^2+9}}\,dx = \int_{u=9}^{u=25} \frac{\frac{1}{2}du}{\sqrt{u}} = \sqrt{u}\Big]_9^{25} = 5-3=2$$

(b) $\displaystyle\int x \sin x\,dx = -x\cos x + \int \cos x\,dx$

$u = x$ $dv = \sin x\,dx$

$du = dx$ $v = -\cos x$

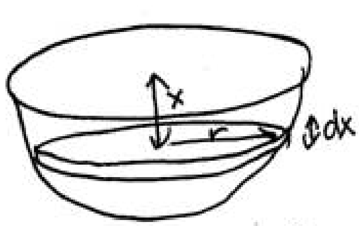
$= -x\cos x + \sin x + C$

(22) $\displaystyle\frac{1}{10}\int_0^{10} P(t)\,dt = \frac{1}{10}\int_0^{10} \left(100 + 10t + (.02)t^2\right)dt$

$= \frac{1}{10}\left(1506.6667\right) = 150.6667$

(23) Ill-posed problem: R is unbounded

(24)



Note $x^2 + r^2 = 8^2$

Volume of slice $= \pi r^2\,dx$

Weight of slice $= 62.5\,(\pi r^2 dx)$

Work of slice $= 62.5x\,(\pi r^2 dx)$

Total work $= \displaystyle\int_0^8 62.5x\,(\pi r^2 dx) = \int_0^8 62.5x(\pi)(64-x^2)\,dx$

$= 64{,}000\pi$ ft-lb

(25) (a)

(b) $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}\left(4 - [2+2\cos\theta]^2\right)d\theta$

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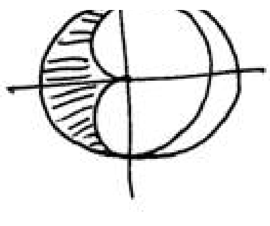

$$= 8 - \pi$$

26 a

n	x_n	y_n	$y_n' = y_n + 1$	$y_{n+1} = y_n + h y_n'$
0	0	1	2	1.5
1	.25	1.5	2.5	2.125
2	.5	2.125		

$$y(1/2) \simeq 2.125$$

b

$$\frac{dy}{y+1} = dx$$

$$\int \frac{dy}{y+1} = \int dx$$

$$\ln|y+1| = x + C$$

$$y(0) = 1$$

$$\ln|1+1| = 0 + C \rightarrow C = \ln 2$$

$$\ln|y+1| = x + \ln 2$$

$$|y+1| = e^{x+\ln 2} = 2e^x$$

$$y = -1 \pm 2e^x$$

Choose $y = -1 + 2e^x$ so that $y(0) = 1$.

27 a Note the equation given does not match the verbal description; should be:

$$\frac{dy}{dt} = ky.$$

$$\frac{dy}{y} = k\,dt; \quad \ln|y| = kt + C; \quad y = \pm e^{kt+c} = \pm e^c e^{kt}$$

$$600 = y(0) = \pm e^c \rightarrow \pm e^c = 600$$

$$y = 600\, e^{kt}$$

$$1800 = y(2) = 600 e^{2k}; \quad k = \tfrac{1}{2} \ln \frac{1800}{600} = \tfrac{1}{2}\ln 3$$

$$\tfrac{1}{2}\ln 3\, t \qquad 600\,(3^{t/2})$$

$$y = 600e^{\frac{1}{2}\ln 3\, t} = 600\left(3^{t/2}\right)$$

(b)
$$5400 = 600\left(3^{t/2}\right)$$
$$3^{t/2} = 9\ ; \quad t/2 = 2\ ; \quad t = 4 \text{ hours}$$

(28) (a)
$$10\frac{dQ}{dt} + \frac{Q}{.02} = 50$$
$$\frac{dQ}{dt} + \frac{Q}{.2} = 5$$
$$\frac{dQ}{dt} + 5Q = 5$$

(b)
$$I = e^{\int 5\,dt} = e^{5t}$$
$$e^{5t}\frac{dQ}{dt} + 5e^{5t}Q = 5e^{5t}$$
$$\frac{d}{dt}\left(e^{5t}Q\right) = 5e^{5t}$$
$$e^{5t}Q = \int 5e^{5t}\,dt = e^{5t} + C$$
$$0 = Q(0) : \quad 0 = e^{5\cdot 0} + C \rightarrow C = -1$$
$$e^{5t}Q = e^{5t} - 1$$
$$Q = 1 - e^{-5t}$$

(29) (a)
$$1 + x^4 + \frac{x^8}{2!} + \frac{x^{12}}{3!}$$

(b)
$$\int_0^{.5}\left(1 + x^4 + \frac{x^8}{2!} + \frac{x^{12}}{3!}\right)dx$$

$$= \left(x + \frac{x^5}{5} + \frac{x^9}{9(2!)} + \frac{x^{13}}{13(3!)} \right) \Big]_0^{.5}$$

$$= .50636$$

30 (a) $4(-1) - 3(-2) + 1(2) = 4$

(b) $\cos^{-1} \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \cos^{-1} \frac{4}{\sqrt{26}\sqrt{9}} = 1.3062 \text{ rad.}$

(c) $\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} = \frac{4}{\sqrt{26}}$

(d) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -3 & 1 \\ -1 & -2 & 2 \end{vmatrix} = -4\vec{i} - 9\vec{j} - 11\vec{k}$

$\text{Area} = \|\vec{a} \times \vec{b}\| = \sqrt{16 + 81 + 121} \doteq 14.765$