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SECTION:

CALCULUS II FINAL EXAM SM122, SM122A  
0755-1055 Wednesday 3 May 2006

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PART ONE: Multiple choice (50%). The first 20 problems are multiple-choice. Fill in the best answer on the bubble sheet. **Write your name, alpha code, and section on your bubble sheet and bubble in your alpha code.** There is no penalty for wrong answers on multiple-choice. Show all your scratch work on this test.

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1. What is the value of  $g'(10)$  if

$$g(x) = \int_1^x \ln(2t^3 + t - 1) dt?$$

- a) 0                      b)  $1/2009$                       c)  $601/2009$                       d)  $\ln(2009)$                       e)  $601 \ln(2009)$
- 

2. Use Euler's method with step size 0.25 to estimate  $y(1)$ , where  $y(x)$  is the solution to the initial-value problem  $y' = 5x + y^2$ ,  $y(0) = 0$ . The answer is closest to:

- a) 2.1307                      b) 1.0429                      c) 1.5219                      d) 4.5157                      e) 3.6127
- 

3. Which of the following is a solution to the differential equation

$$\frac{dy}{dx} - 4y = 0?$$

- a)  $y = e^{-4x}$                       b)  $y = e^{4x}$                       c)  $y = e^{2x^2}$                       d)  $y = \sin(2x)$                       e)  $y = 2x^2$
- 

4. Which of the following pairs of polar coordinates  $(r, \theta)$  does *not* correspond to the point with Cartesian coordinates  $(-\sqrt{2}, \sqrt{2})$ ?

- a)  $(-2, -\pi/4)$                       b)  $(2, 3\pi/4)$                       c)  $(-2, -3\pi/4)$                       d)  $(2, -5\pi/4)$                       e)  $(-2, 7\pi/4)$

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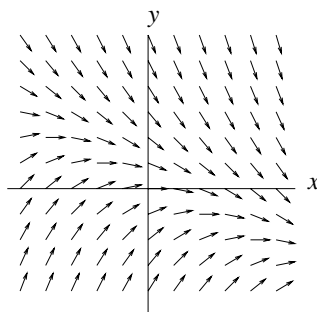
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5. The direction field above is a direction field for a differential equation of the form

$$y' = ax + by.$$

The values of  $a$  and  $b$  satisfy:

- a)  $a > 0, b > 0$ ,      b)  $a > 0, b < 0$       c)  $a < 0, b > 0$       d)  $a = 0, b = 0$       e)  $a < 0, b < 0$
- 

For Problems 6 and 7, consider a continuous function  $g(x)$  such that

$$g(0) = 2, \quad g(2) = 5, \quad g(4) = 4, \quad \int_0^2 g(x) dx = 7, \quad \text{and} \quad \int_0^4 g(x) dx = 13.$$

6. Compute

$$\int_0^2 x g(x^2) dx.$$

- a)  $7/2$                       b)  $13/2$                       c)  $7$                       d)  $13$                       e)  $14$

7. Compute

$$\int_0^2 x g'(x) dx.$$

- a)  $3$                       b)  $5$                       c)  $7$                       d)  $9$                       e) Cannot be computed from the information given.

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8. The average value of the function  $f(x) = \sin(3x)$  on the interval  $[0, \pi/3]$  is:

- a)  $3/2$                       b)  $1/\pi$                       c)  $0$                       d)  $2/3$                       e)  $2/\pi$
- 

9. Suppose  $f$  is a continuous function such that  $2 < f(x) < 3$  for all  $x$ . If the region between the  $x$ -axis and the graph of  $f$  for  $0 < x < 4$  is revolved around the  $x$ -axis, the volume of the resulting solid is:

- a) less than 10  
b) between 10 and 28  
c) between 28 and 50  
d) between 50 and 120  
e) between 120 and 180.
- 

10. The substitution  $u = x^2$  transforms the integral

$$\int \frac{x}{1+x^2} dx \quad \text{to:}$$

- a)  $\int \frac{2}{1+u} du$     b)  $\int \frac{u}{1+u} du$     c)  $\int \frac{u}{2(1+u)} du$     d)  $\int \frac{1}{2(1+u)} du$     e)  $\int \frac{2u}{1+u} du$
- 

11. The integral

$$\int_0^{\infty} e^{-x} dx.$$

- a) has a value of 0  
b) has a value of 1  
c) diverges  
d) has value infinity  
e) none of the above.
- 

12. For what value of  $a$  is  $y = \sin(ax)$  a solution to the differential equation

$$y'' + 4y = 0?$$

- a)  $-1$                       b)  $1$                       c)  $2$                       d)  $3$                       e)  $4$

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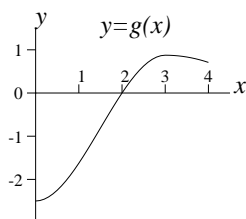
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Problems 13 and 14 concern a function  $g(x)$  whose graph is as shown:

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13. Define

$$G(x) = \int_0^x g(t) dt.$$

Then  $G$  is increasing on what interval?

- a) (0,2)                      b) (0,3)                      c) (0,4)                      d) (2,3)                      e) (2,4)

14. The same function  $G$  is concave up on what interval?

- a) (0,2)                      b) (0,3)                      c) (0,4)                      d) (2,3)                      e) (2,4)
- 

15. Which integral is best done using partial fractions?

- a)  $\int xe^x dx$                       b)  $\int [1/(x^2 + x)] dx$                       c)  $\int \cos(x^2) dx$                       d)  $\int \ln x dx$                       e)  $\int [1/(x^2 + 1)] dx$
- 

16. A new radioactive substance is discovered in the Z-burgers in King Hall. It is found that 90% of this substance decays over a period of 9 years. The half-life of the substance is closest to:

- a) 1 year                      b) 2 years                      c) 5 years                      d) 10 years                      e) a million zillion kajillion years

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17. A vector perpendicular to both  $\langle 2, 1, -2 \rangle$  and  $\langle 3, 0, -1 \rangle$  is:

- a)  $5\mathbf{i} + 5\mathbf{k}$                       b)  $\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$                       c)  $\langle 2, 2, 6 \rangle$                       d)  $\sqrt{2}$                       e) None of these.

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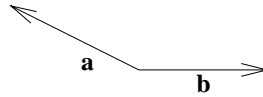
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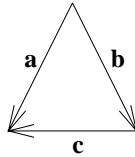
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18. For vectors  $\mathbf{a}$  and  $\mathbf{b}$  as shown,  $\mathbf{a} \cdot \mathbf{b}$  is:

- a) positive      b) negative      c) zero      d) imaginary      e) undefined
- 



19. For vectors  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  as shown, which is true?

- a)  $\mathbf{a} + \mathbf{b} = \mathbf{c}$       b)  $\mathbf{b} + \mathbf{c} = \mathbf{a}$       c)  $\mathbf{c} + \mathbf{a} = \mathbf{b}$       d)  $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$       e)  $\mathbf{a} \times \mathbf{b} = \mathbf{c}$
- 

20. A simple series circuit contains a constant 10V EMF, an inductor with inductance 7 henries, and a 2- $\Omega$  resistor. If the current  $I$  is constant, then  $I$  equals:

- a) 2 amps      b) 10/7 amps      c) 2/7 amps      d) 1/5 amp      e) 5 amps

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PART TWO: Longer answers (50%). Work the following ten problems. They are not multiple choice. Show all work and your answers on this test paper.

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21. Use a Riemann sum with 6 subintervals to estimate

$$\int_0^3 (2x^2 - 1) dx.$$

Use left endpoints of subintervals as sample points.

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22. Find the area of the region bounded by the curves  $y = x^2$  and  $x = y^2$ .

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23. A simple series electric circuit has a switch, a 0.005 farad capacitor, a  $50 \Omega$  resistor, and a power supply which a electromotive force (EMF) given by  $E(t) = 100e^{-4t}$  Volts. Initially, there is no charge on the capacitor.

a. Formulate an initial value problem (IVP) that models this circuit.

b. Let  $Q(t)$  represent the charge on the capacitor at time  $t$ . Solve the initial value problem in part (a) to find an expression for the charge on the capacitor at time  $t$ .

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24. The region bounded by the curves  $y = x$ ,  $y = 2x$ , and  $y = 4$  is revolved around the  $y$ -axis. Find the volume of the resulting solid.

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25. Each anchor on the U.S.S. Eisenhower weighs 60,000 pounds, with an anchor chain that weighs 625 pounds per foot (underwater). How much work is done in hoisting one anchor to the surface from a depth of 1000 feet directly beneath the ship?

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26. A spring stretches one foot beyond its natural position under a force of 100 pounds. How much work (in foot-pounds) is done in stretching the spring 3 feet beyond its natural position?



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27. In this problem we deal with the three points  $P = (3, 0, 0)$ ,  $Q = (0, 2, 0)$ , and  $R = (0, 0, 1)$ .

a) Compute the dot product  $\vec{PQ} \cdot \vec{PR}$ .

b) Compute the cross product  $\vec{PQ} \times \vec{PR}$ .

c) Compute the angle  $\angle QPR$  to the nearest degree.

d) Compute the area of triangle  $PQR$ .

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28. a) Convert the equation  $r = 4 \sin \theta$  to Cartesian (rectangular) coordinates and sketch its graph.

b) Convert the equation  $x^2 + y^2 = x - 4y$  to polar coordinates.

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29. a) Find the general solution to the differential equation

$$y' + \frac{2}{x}y = 5x^2.$$

b) Find the particular solution satisfying the initial condition  $y(1) = 3$ .

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30. A bacterial culture starts with 420 bacteria and grows at a rate proportional to its size. After 7 hours there are 6900 bacteria.

a) Find a formula for the population at time  $t$  hours.

b) When will the population reach 29,000?