**NAME:** **ALPHA:**
**INSTRUCTOR:** **SECTION:**

**CALCULUS II FINAL EXAM SM122, SM122A** 

**0755-1055 Wednesday 3 May 2006**

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

---

PART ONE: Multiple choice (50%). The first 20 problems are multiple-choice. Fill in the best answer on the bubble sheet. **Write your name, alpha code, and section on your bubble sheet and bubble in your alpha code.** There is no penalty for wrong answers on multiple-choice. Show all your scratch work on this test.

---

1. What is the value of $g'(10)$ if

$$g(x) = \int_1^x \ln(2t^3 + t - 1)\,dt?$$

$g'(x) = \ln(2x^3 + x - 1)$
$g'(10) = \ln(2009)$

a) 0 b) 1/2009 c) 601/2009 d) ln(2009) e) 601 ln(2009)

---

2. Use Euler's method with step size 0.25 to estimate $y(1)$, where $y(x)$ is the solution to the initial-value problem $y' = 5x + y^2$, $y(0) = 0$. The answer is closest to:

a) 2.1307 b) 1.0429 c) 1.5219 d) 4.5157 e) 3.6127

| $n$ | $x_n$ | $y_n$ | $y_n'$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | .25 | 0 | 1.25 | .3125 |
| 2 | .5 | .3125 | 2.59766 | .961914 |
| 3 | .75 | .961914 | 4.67528 | 2.13073 |

---

3. Which of the following is a solution to the differential equation

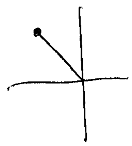
$$\frac{dy}{dx} - 4y = 0?$$

a) $y = e^{-4x}$ b) $y = e^{4x}$ c) $y = e^{2x^2}$ d) $y = \sin(2x)$ e) $y = 2x^2$

---

4. Which of the following pairs of polar coordinates $(r, \theta)$ does *not* correspond to the point with Cartesian coordinates $(-\sqrt{2}, \sqrt{2})$ ?

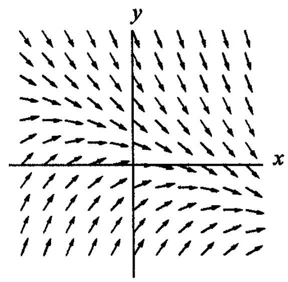
a) $(-2, -\pi/4)$ b) $(2, 3\pi/4)$ c) $(-2, -3\pi/4)$ d) $(2, -5\pi/4)$ e) $(-2, 7\pi/4)$



NAME: ALPHA:
INSTRUCTOR: SECTION:

CALCULUS II FINAL EXAM SM122, SM122A 
0755-1055 Wednesday 3 May 2006

CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST




5. The direction field above is a direction field for a differential equation of the form

$$y' = ax + by.$$

The values of $a$ and $b$ satisfy:

a) $a > 0, b > 0,$ b) $a > 0, b < 0$ c) $a < 0, b > 0$ d) $a = 0, b = 0$ e) $a < 0, b < 0$

For Problems 6 and 7, consider a continuous function $g(x)$ such that

$$g(0) = 2, \quad g(2) = 5, \quad g(4) = 4, \quad \int_0^2 g(x)\,dx = 7, \quad \text{and} \quad \int_0^4 g(x)\,dx = 13.$$

6. Compute

$$\int_0^2 x\,g(x^2)\,dx.$$

a) 7/2 b) 13/2 c) 7 d) 13 e) 14

$u = x^2$
$du = 2x\,dx$ $\int_0^4 \frac{1}{2} g(u)\,du$

7. Compute

$$\int_0^2 x\,g'(x)\,dx.$$

a) 3 b) 5 c) 7 d) 9 e) Cannot be computed from the information given.

$u = x \quad dv = g'(x)dx$
$du = dx \quad v = g(x)$

$x g(x) \Big]_0^2 - \int_0^2 g(x)\,dx$

$10 - 0 - 7$

**CALCULUS II FINAL EXAM SM122, SM122A**


**0755-1055 Wednesday 3 May 2006**

**CALCULUS ALLOWED—SHOW ALL WORK ON THIS TEST**

8. The average value of the function $f(x) = \sin(3x)$ on the interval $[0, \pi/3]$ is:
a) 3/2 b) $1/\pi$ c) 0 d) 2/3 e) $2/\pi$

$\frac{3}{\pi}\int_0^{\pi/3} \sin(3x)\,dx =$

9. Suppose $f$ is a function such that $2 < f(x) < 3$ for all $x$. If the region between the $x$-axis and the graph of $f$ for $0 < x < 4$ is revolved around the $x$-axis, the volume of the resulting solid is:
a) less than 10
b) between 10 and 28
c) between 28 and 50
d) between 50 and 120
e) between 120 and 180.

10. The substitution $u = x^2$ transforms the integral

$$\int \frac{x}{1+x^2}\,dx \quad \text{to:}$$

$du = 2x\,dx$ $\int \frac{du}{2(1+u)}$

a) $\int \frac{2}{1+u}\,du$ b) $\int \frac{u}{1+u}\,du$ c) $\int \frac{u}{2(1+u)}\,du$ d) $\int \frac{1}{2(1+u)}\,du$ e) $\int \frac{2u}{1+u}\,du$

11. The integral

$$\int_0^\infty e^{-x}\,dx. = \lim_{t\to\infty}\int_0^t e^{-t}\,dt$$

$$= \lim_{t\to\infty} -e^{-x}\Big]_0^t = \lim_{t\to\infty}(1-e^{-t}) = 1$$

a) has a value of 0
b) has a value of 1
c) diverges
d) has value infinity
e) none of the above.

12. For what value of $a$ is $y = \sin(ax)$ a solution to the differential equation

$$y'' + 4y = 0?$$

a) $-1$ b) 1 c) 2 d) 3 e) 4

$y = \sin(ax)$ $y' = a\cos(ax)$ $y'' = -a^2\sin(ax)$

$-a^2\sin(ax) + 4\sin(ax) = 0 \rightarrow a = \pm 2$

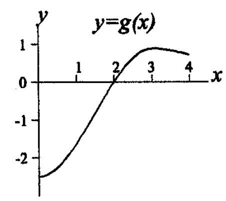
**NAME:** **ALPHA:**
**INSTRUCTOR:** **SECTION:**



**0755-1055 Wednesday 3 May 2006**

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

Problems 13 and 14 concern a function $g(x)$ whose graph is as shown:




13. Define

$$G(x) = \int_0^x g(t)\,dt.$$

Then $G$ is increasing on what interval? Need $G' = g$ positive

a) (0,2) b) (0,3) c) (0,4) d) (2,3) e) (2,4)

14. The same function $G$ is concave up on what interval? Need $G' = g$ increasing

a) (0,2) b) (0,3) c) (0,4) d) (2,3) e) (2,4)

15. Which integral is best done using partial fractions?

a) $\int xe^x\,dx$ b) $\int [1/(x^2+x)]\,dx$ c) $\int \cos(x^2)\,dx$ d) $\int \ln x\,dx$ e) $\int [1/(x^2+1)]\,dx$

16. A new radioactive substance is discovered in the Z-burgers in King Hall. It is found that 90% of this substance decays over a period of 9 years. The half-life of the substance is closest to:

a) 1 year b) 2 years c) 5 years d) 10 years e) a million zillion kajillion years

$e^{9k} = .1$ $e^{kt} = .5$

$k = \frac{1}{9}\ln .1$ $t = \frac{1}{k}\ln(.5) = 9\,\frac{\ln .5}{\ln .1} = 2.70927$ years

17. A vector perpendicular to both $\langle 2, 1, -2\rangle$ and $\langle 3, 0, -1\rangle$ is:

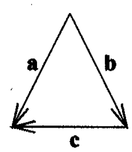
a) $5\mathbf{i} + 5\mathbf{k}$ b) $\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ c) $\langle 2, 2, 6\rangle$ d) $\sqrt{2}$ e) None of these.

$$\langle 2,1,-2\rangle \times \langle 3,0,-1\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ 3 & 0 & -1 \end{vmatrix} = \langle -1,-4,-3\rangle$$

CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST


18. For vectors **a** and **b** as shown, $\mathbf{a} \cdot \mathbf{b}$ is:

a) positive b) negative c) zero d) imaginary e)undefined




19. For vectors **a**, **b**, and **c** as shown, which is true?

a) $\mathbf{a}+\mathbf{b}=\mathbf{c}$ b) $\mathbf{b}+\mathbf{c}=\mathbf{a}$ c) $\mathbf{c}+\mathbf{a}=\mathbf{b}$ d) $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$ e) $\mathbf{a}\times\mathbf{b}=\mathbf{c}$

20. A simple series circuit contains a constant 10V EMF, an inductor with inductance 7 henries, and a 2-Ω resistor. If the current $I$ is constant, then $I$ equals:

a) 2 amps b) 10/7 amps c) 2/7 amps d) 1/5 amp e) 5 amps

$$7\frac{dI}{dt} + 2I = 10$$

$$2I = 10$$

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

PART TWO: Longer answers (50%). Work the following ten problems. They are not multiple choice. Show all work and your answers on this test paper.

21.

a) Use a Riemann sum with 6 subintervals to estimate

$$\int_0^3 (2x^2 - 1)\, dx.$$

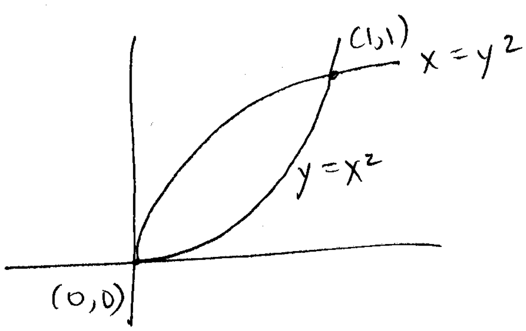
Use left endpoints of subintervals as sample points.

| $x$ | $f(x)$ |
|---|---|
| 0 | -1 |
| .5 | -.5 |
| 1 | 1 |
| 1.5 | 3.5 |
| 2 | 7 |
| 2.5 | 11.5 |

$.5(-1 - .5 + 1 + 3.5 + 7 + 11.5)$

$= 10.75$

22. Find the area of the region bounded by the curves $y = x^2$ and $x = y^2$.




$$\int_0^1 (\sqrt{x} - x^2)\, dx$$

$= 1/3$

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

23. A simple series electric circuit has a switch , a 0.005 farad capacitor, a 50 $\Omega$ resistor, and a power supply which a electromotive force (EMF) given by $E(t) = 100e^{-4t}$ Volts. Initially, there is no charge on the capacitor.

a. Formulate an initial value problem (IVP) that models this circuit.

$$50\frac{dQ}{dt} + \frac{Q}{.005} = 100e^{-4t}, \quad Q(0)=0$$

$$\frac{dQ}{dt} + 4Q = 2e^{-4t}$$

b. Let $Q(t)$ represent the charge on the capacitor at time $t$. Solve the initial value problem in part (a) to find an expression for the charge on the capacitor at time $t$.

$$I = e^{\int 4dt} = e^{4t}$$

$$e^{4t}\frac{dQ}{dt} + 4e^{4t}Q = 2$$

$$\frac{d}{dt}(e^{4t}Q) = 2$$

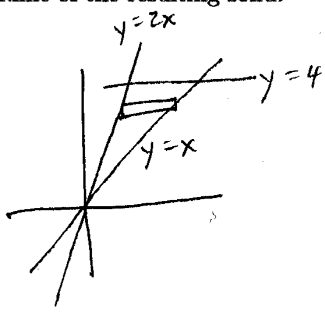
$$e^{4t}Q = 2t + C$$

$$Q = 2te^{-4t} + Ce^{-4t}$$

$$0 = Q(0) = C \longrightarrow C = 0$$

$$Q = 2te^{-4t}$$

24. The region bounded by the curves $y = x$, $y = 2x$, and $y = 4$ is revolved around the $y$-axis. Find the volume of the resulting solid.



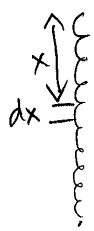

$$\int_0^4 \pi\left[y^2 - \left(\frac{y}{2}\right)^2\right]dy$$

$$= 16\pi$$

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

---

25. Each anchor on the U.S.S. Eisenhower weighs 60,000 pounds, with an anchor chain that weighs 625 pounds per foot. How much work is done in hoisting one anchor to the surface from a depth of 1000 feet directly beneath the ship?




chain

$$\int_0^{1000} x\,(625\,dx) = 312{,}500{,}000$$

anchor $1000 \times 60000 = 60000000$

total: 372,500,000 ft-lb.

---

26. A spring stretches one foot beyond its natural position under a force of 100 pounds. How much work (in foot-pounds) is done in stretching the spring 3 feet beyond its natural position?

$F = kx$

$100 = k(1)$

$k = 100$

$$\int_0^3 100x\,dx = 450 \text{ ft-lb.}$$

**NAME:** **ALPHA:**
**INSTRUCTOR:** **SECTION:**

**CALCULUS II FINAL EXAM SM122, SM122A**



**0755-1055 Wednesday 3 May 2006**

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

27. In this problem we deal with the three points $P = (3,0,0)$, $Q = (0,2,0)$, and $R = (0,0,1)$.

a) Compute the dot product $\vec{PQ} \cdot \vec{PR}$.

$\vec{PQ} = \langle -3, 2, 0 \rangle \qquad \vec{PR} = \langle -3, 0, 1 \rangle$

$\vec{PQ} \cdot \vec{PR} = 9$

b) Compute the cross product $\vec{PQ} \times \vec{PR}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 0 \\ -3 & 0 & 1 \end{vmatrix} = 2\vec{i} + 3\vec{j} + 6\vec{k}$$

c) Compute the angle $\angle QPR$ to the nearest degree.

$$\cos^{-1} \frac{\vec{PQ} \cdot \vec{PR}}{\|\vec{PQ}\| \, \|\vec{PR}\|} = \cos^{-1} \frac{9}{\sqrt{(13)(10)}} = .661043 \text{ rad}$$

$$= 37.875^\circ$$

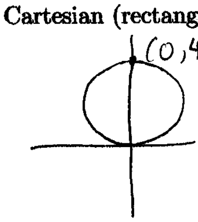
d) Compute the area of triangle $PQR$.

$$\frac{1}{2} \| \vec{PQ} \times \vec{PR} \| = \frac{1}{2} \sqrt{4+9+36} = \frac{7}{2}$$

28. a) Convert the equation $r = 4\sin\theta$ to Cartesian (rectangular) coordinates and sketch its graph.

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$




b) Convert the equation $x^2 + y^2 = x - 4y$ to polar coordinates.

$r^2 = r\cos\theta - 4r\sin\theta$

$r = \cos\theta - 4\sin\theta$

**CALCULATORS ALLOWED—SHOW ALL WORK ON THIS TEST**

29. a) Find the general solution to the differential equation

$$y' + \frac{2}{x}y = 5x^2.$$

Linear.

$I = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

$x^2 y' + 2xy = 5x^4$

$(x^2 y)' = 5x^4$

$x^2 y = x^5 + C$

$y = x^3 + Cx^{-2}$

b) Find the particular solution satisfying the initial condition $y(1) = 3$.

$3 = y(1) = 1 + C \rightarrow C = 2$

$y = x^3 + 2x^{-2}$

30. A bacterial culture starts with 420 bacteria and grows at a rate proportional to its size. After 7 hours there are 6900 bacteria.

a) Find a formula for the population at time $t$ hours.

$P = Ce^{kt}$

$P(0) = C = 420$

$P(7) = 420e^{7k} = 6900$

$k = \frac{1}{7}\ln\frac{6900}{420} = .39986$

$P = 420e^{.39986t}$

b) When will the population reach 29,000?

$29000 = 420e^{.39986t}$

$t = \frac{1}{.39986}\ln\frac{29000}{420} = 10.5907$ hours