NAME: ALPHA NUMBER:
INSTRUCTOR: SECTION:

SPRING 2006-2007 FINAL EXAMINATION: CALCULUS II (SM122, SM122A)

0755-1055 Tuesday 01 May 2007

CALCULATORS ALLOWED – SHOW ALL WORK IN THIS TEST PACKAGE

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on the Scantron bubble sheet. Write your name, alpha number, and section on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test (using the back of pages if necessary).

---

1. A boat traveling in a straight line has velocities at 15-second intervals given by the following table.

t (s)	0	15	30	45	60
v (ft/s)	10	12	15	14	18

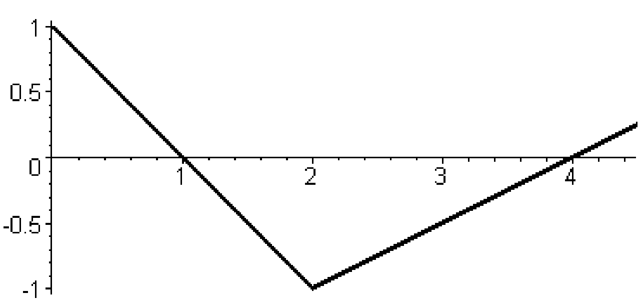
Use left endpoints with $n = 4$ to approximate the distance traveled:

a) 765 ft
b) 825 ft
c) 885 ft
d) 1035 ft
e) none of the above

---

2. The graph of $y = f(x)$ is given below and consists of two line segments. For $g(x) = \int_0^x f(t)dt$, what is $g(4)$?

a) -2
b) $-3/2$
c) -1
d) $-1/2$
e) 0

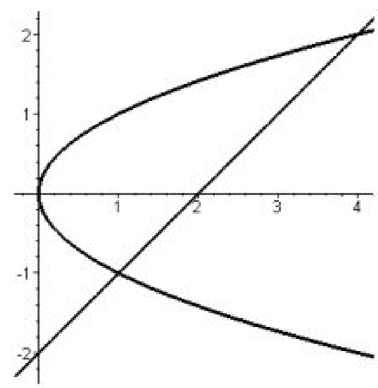


---

3. If $f = g + h$ where g is an odd function and h is an even function (all continuous), then $\int_{-a}^{a} f(x)dx =$

a) 0
b) $2\int_0^a g(x)\,dx$
c) $2\int_0^a h(x)\,dx$
d) $2\int_0^a \left(g(x) + h(x)\right)dx$
e) None of the above

4. For the region bounded by the two curves with equations $x = y^2$ and $y = x - 2$ as drawn to the right, the area is given by which integral?



a) $\int_0^4 \left(\sqrt{x} - (x-2)\right) dx$

b) $\int_0^4 \left(x - 2 - \sqrt{x}\right) dx$

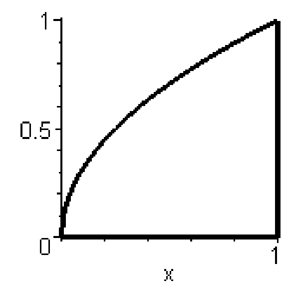
c) $\int_{-1}^2 \left(y^2 - (y+2)\right) dy$

d) $\int_{-1}^2 \left(y + 2 - y^2\right) dy$

e) $\int_{-1}^2 \left(y^2 + y + 2\right) dy$

---

5. Find the volume of the solid of revolution formed by rotating the region under the curve $y = \sqrt{x}$ from 0 to 1 about the x-axis.



a) $2/3$
b) π
c) $\pi/3$
d) $2\pi/3$
e) $\pi/2$

---

6. Which of the following is closest to the amount of work it takes to pump all the water out of the top of a full cubical container 2 meters on each edge? (The acceleration due to gravity is $g = 9.8\,\text{m/sec}^2$ and the mass density of water is 1000 kg/m^3.)

a) 39,200 J
b) 78,400 J
c) 117,600 J
d) 156,800 J
e) 196,000 J

---

7. One application of integration by parts to $\int x^3 e^{2x}\,dx$ can change it to which of the following?

a) $\frac{1}{2}x^3e^{2x} - \frac{3}{2}\int x^2 e^{2x}\,dx$

b) $\frac{1}{2}x^3e^{2x} + \frac{3}{2}\int x^2 e^{2x}\,dx$

c) $\frac{1}{2}x^3e^{2x} - 3\int x^2 e^{2x}\,dx$

d) $\frac{1}{2}x^3e^{2x} + 3\int x^2 e^{2x}\,dx$

e) None of the above

8. Using the trig substitution $x = 3\sin(\theta)$ changes $\int_0^{3/2} \frac{x^2}{\sqrt{9-x^2}}dx$ to which of the following?

a) $9\int_0^{\pi/3} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

b) $9\int_0^{\pi/6} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

c) $9\int_0^{3/2} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

d) $9\int_0^{\pi/3} \sin^2(\theta)d\theta$

e) $9\int_0^{\pi/6} \sin^2(\theta)d\theta$

---

9. Noting that $\int e^{-x}(\sin(x)+\cos(x))dx = -e^{-x}\cos(x) + C$, the improper integral $\int_0^{\infty} e^{-x}(\sin(x)+\cos(x))dx$ equals:

a) $\lim_{T\to 0^+}(-e^{-T}\cos(T)+1)$

b) $\lim_{T\to 0^-}(-e^{-T}\cos(T)+1)$

c) $\lim_{T\to \infty}(-e^{-T}\cos(T)+1)$

d) $\lim_{T\to \infty}(-e^{-T}\cos(T))$

e) None of the above

---

10. If a lamina has mass and moments given by: $m = 3, M_x = 5, M_y = 7$, then the center of mass, $(\bar{x}, \bar{y})$, equals:

a) $\left(\frac{3}{5}, \frac{3}{7}\right)$

b) $\left(\frac{3}{7}, \frac{3}{5}\right)$

c) $\left(\frac{5}{3}, \frac{7}{3}\right)$

d) $\left(\frac{7}{3}, \frac{5}{3}\right)$

e) None of the above

---

11. The function given by $y = 3\sin(2x)$ is a solution to which of the following second order initial-value problems?

a) $y'' = -y, \quad y(0) = 0, \ y'(0) = 0$

b) $y'' = 4y, \quad y(0) = 0, \ y'(0) = 6$

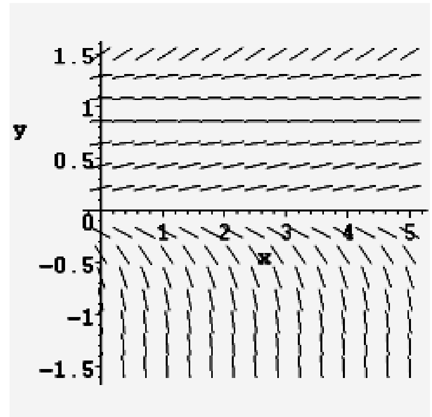
c) $y'' = -y, \quad y(0) = 0, \ y'(0) = 3$

d) $y'' = -3y, \quad y(0) = 0, \ y'(0) = 6$

e) $y'' = -4y, \quad y(0) = 0, \ y'(0) = 6$

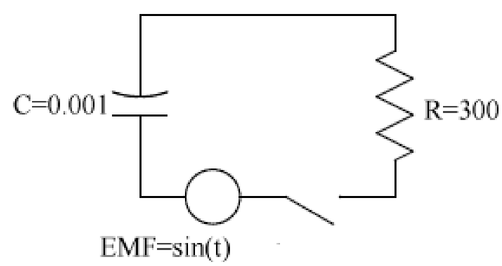
12. To the right is the direction field for differential equation $y' = y(y-1)^2$. For which one of the initial values $y(0) = y_0$ listed below will the limiting behavior of the solution approach 1 (so that $\lim_{x\to\infty} y(x) = 1$)?

a) $y_0 = 1.5$
b) $y_0 = 0.5$
c) $y_0 = 0$
d) $y_0 = -0.5$
e) $y_0 = -1.0$



13. A certain isotope decays so that the remaining mass after t days is given by $m = m_0 e^{-0.12t}$ where m_0 is the initial mass. The half life is closest to which of the following?
a) 0.12 days
b) 0.83 days
c) 2.3 days
d) 5.8 days
e) 8.3 days

14. Once the switch is closed, a differential equation that describes the circuit to the right is
a) $1000Q' + 300Q = \sin(t)$
b) $300Q' + 1000Q = \sin(t)$
c) $Q'/1000 + 300Q = \sin(t)$
d) $Q'/300 + 1000Q = \sin(t)$
e) $Q'/300 + Q/1000 = \sin(t)$



15. The point with Cartesian coordinates $(x, y) = (2,2)$ can have which of the following for polar coordinates (r,θ)?
a) $(2,\pi/4)$
b) $(2,-\pi/4)$
c) $(2\sqrt{2},-\pi/4)$
d) $(2\sqrt{2},\pi/4)$
e) $(4\sqrt{2},-\pi/4)$

---

16. Find the limit of the sequence given by $a_n = \dfrac{3-5n^2}{2n+n^2}$

a) $-\infty$
b) -5
c) $-5/2$
d) $3/2$
e) $5/2$

---

17. The 5th degree Taylor polynomial, $T_5(x)$, at $a = 0$ (also called a Maclaurin polynomial) for the sine function, $f(x) = \sin(x)$, is:

a) $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6} + \dfrac{x^4}{24} + \dfrac{x^5}{120}$
b) $1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} - \dfrac{x^5}{120}$
c) $1 - \dfrac{x^2}{2} + \dfrac{x^4}{24}$
d) $-x + \dfrac{x^3}{6} - \dfrac{x^5}{120}$
e) $x - \dfrac{x^3}{6} + \dfrac{x^5}{120}$

---

18. The sum of vectors $\mathbf{a} = \langle 1,-2,4 \rangle$ and $\mathbf{b} = \langle 0,2,1 \rangle$ in terms of standard basis vectors, is:

a) $-4\mathbf{j} + 4\mathbf{k}$
b) $\mathbf{i} + 5\mathbf{k}$
c) $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
d) $\mathbf{i} + 5\mathbf{j}$
e) $\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

---

19. The scalar projection of vector $\mathbf{b} = \langle 3,4 \rangle$ onto vector $\mathbf{a} = \langle 5,12 \rangle$ is:

a) 63
b) $63/5$
c) $63/13$
d) $5/63$
e) $13/63$

---

20. The cross product $\mathbf{k} \times (2\mathbf{j})$ equals:

a) $2\mathbf{i}$
b) $-2\mathbf{i}$
c) $2\mathbf{j} + \mathbf{k}$
d) $2\mathbf{j} - \mathbf{k}$
e) the zero vector

PART TWO: FREE RESPONSE (50%). The remaining 10 problems are not multiple choice. Answer them on this test paper in the blank space provided. If space is insufficient use the backs of the pages. Show the details of your work and box your answers.

21. Find the volume created by rotating the region bounded by $y = x$ and $y = x^2$ about the *y*-axis both

a) by the method of washers and

b) the method of cylindrical shells (showing that you get the same value).

22. Suppose the work required to stretch a spring 2 ft beyond its natural length is 12 ft-lb.

a) Find the spring constant *k* for this spring.

b) How much work is needed to stretch a spring with the same spring constant 3 ft beyond its natural length?

---

23. a) Find constants A and B so that $\dfrac{12}{x(x+2)} = \dfrac{A}{x} + \dfrac{B}{x+2}$.

b) Evaluate by hand $\int \dfrac{12}{x(x+2)}\,dx$ using part a), showing all work.

c) Consider the series $\sum_{n=1}^{\infty} \dfrac{12}{n(n+2)}$. Express the eighth partial sum, s_8, as a fraction in lowest terms. (Hint: use part a.)

d) Find the sum of the series in part c by taking the limit of the partial sums.

---

24. a) Use substitution to show that the trigonometric integral $\int_0^{\pi} \sin(x)\cos^2(x)\,dx$ equals 2/3.

b) Use Simpson's rule with 4 subintervals to approximate the integral in part a. (Give the answer to 6 decimal places.)

---

25. a) Solve the initial-value problem: $\frac{dy}{dt} = te^{y}$, $y(0) = 0$, giving an explicit solution for y as a function of t.

b) Use Euler's method with $n = 2$ steps to approximate $y(1)$ in the problem in part a).

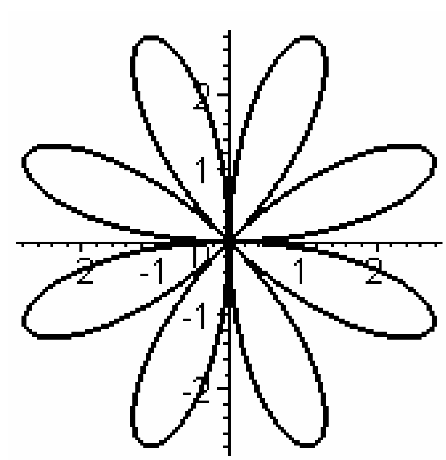
---

26. An R-L circuit with a 2 henry inductor, a 20 ohm resistor, and an EMF of $E(t) = e^{-5t}$ leads to the differential equation for current: $2I' + 20I = e^{-5t}$. Assuming a switch closes the circuit at time 0 (so that $I(0) = 0$), solve for $I(t)$.

27. a) Eliminate the parameter to find a (simplified) Cartesian equation for the parametric curve $x = 4t^2 - 16t + 14$, $y = 4 - 2t$ and use it to sketch the curve.

b) Use the parametric equations in part a) to verify that the tangent line to the curve at $t = 1$ has equation $x - 4y + 6 = 0$

28. Find the area of one "petal" of the graph in polar coordinates of $r = 3\sin(4\theta)$ shown to the right.



---

29. a) Find the unit vector in the direction from point $(2,-1,4)$ to $(3,-2,6)$.

b) Show that the vector found in part a) is orthogonal (perpendicular) to $2\mathbf{i}-\mathbf{k}$.

---

30. Note that the lines with parametric equations $x=2t,\ y=1+t,\ z=-3-t$ and $x=2-t,\ y=2+t,\ z=-4+2t$ both pass through the point $(2,2,-4)$. Find an equation for the plane that contains both lines.

(end)