NAME: **KEY** ALPHA NUMBER:
INSTRUCTOR: SECTION:

SPRING 2006-2007 FINAL EXAMINATION: CALCULUS II (SM122, SM122A)
0755-1055 Tuesday 01 May 2007
CALCULATORS ALLOWED – SHOW ALL WORK IN THIS TEST PACKAGE

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on the Scantron bubble sheet. Write your name, alpha number, and section on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test (using the back of pages if necessary).

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1. A boat traveling in a straight line has velocities at 15-second intervals given by the following table.

t (s)	0	15	30	45	60
v (ft/s)	10	12	15	14	18

Use left endpoints with $n = 4$ to approximate the distance traveled:

a) 765 ft
b) 825 ft
c) 885 ft
d) 1035 ft
e) none of the above

We compute left endpoint approximation as: $15(10+12+15+14) = 765$ a)

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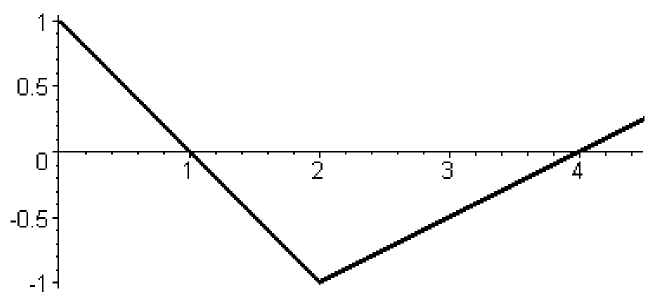
2. The graph of $y = f(x)$ is given below and consists of two line segments. For $g(x) = \int_0^x f(t)dt$, what is $g(4)$?

a) -2
b) $-3/2$
c) -1
d) $-1/2$
e) 0



We break the integral into two pieces. The first is zero by symmetry, the second is a triangle below the axis. So we get: $g(4) = \int_0^2 f(t)\,dt + \int_2^4 f(t)\,dt = 0 + (-1)\frac{1}{2}(1)(2) = -1$ c)

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3. If $f = g + h$ where g is an odd function and h is an even function (all continuous), then $\int_{-a}^{a} f(x)dx =$

a) 0

b) $2\int_0^a g(x)\,dx$

c) $2\int_0^a h(x)\,dx$

d) $2\int_0^a (g(x)+h(x))dx$

e) None of the above

First we use the fact that the integral of a sum is the sum of the integrals. Then we use the properties of integrating odd and even functions over a symmetric interval. This gives us:

$\int_{-a}^{a} f(x)dx = \int_{-a}^{a} g(x)dx + \int_{-a}^{a} h(x)dx = 0 + 2\int_0^a h(x)dx$ c)

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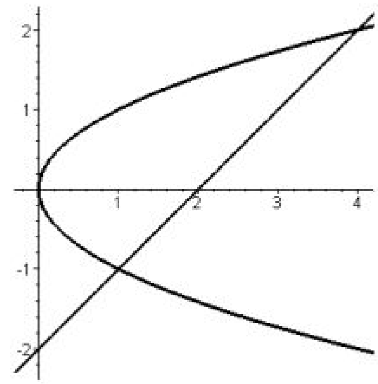
4. For the region bounded by the two curves with equations $x = y^2$ and $y = x - 2$ as drawn to the right, the area is given by which integral?



a) $\int_0^4 \left(\sqrt{x} - (x-2)\right)dx$

b) $\int_0^4 \left(x - 2 - \sqrt{x}\right)dx$

c) $\int_{-1}^{2} \left(y^2 - (y+2)\right)dy$

d) $\int_{-1}^{2} \left(y + 2 - y^2\right)dy$ [boxed]

e) $\int_{-1}^{2} \left(y^2 + y + 2\right)dy$

Since there's a left curve and a right curve with integrate with respect to *y*. The intersection points are $(1,-1)$ and $(4,2)$ so the limits of integration are from $y = -1$ to $y = 2$. The integrand is big minus small (right minus left) so we solve for *x* and subtract the straight line part minus the parabola part. d)

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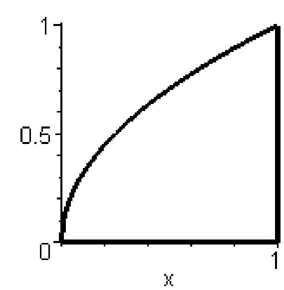
5. Find the volume of the solid of revolution formed by rotating the region under the curve $y = \sqrt{x}$ from 0 to 1 about the *x*-axis.



a) $2/3$

b) π

c) $\pi/3$

d) $2\pi/3$

e) $\pi/2$ [boxed]

Using disks, $V = \int_0^1 \pi\left(\sqrt{x}\right)^2 dx = \int_0^1 \pi\, x dx = \pi\, x^2/2]_0^1 = \pi/2$. e)

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6. Which of the following is closest to the amount of work it takes to pump all the water out of the top of a full cubical container 2 meters on each edge? (The acceleration due to gravity is $g = 9.8\,\text{m/sec}^2$ and the mass density of water is 1000 kg/m^3.)

a) 39,200 J

b) 78,400 J [boxed]

c) 117,600 J

d) 156,800 J

e) 196,000 J

Slicing horizontally, we "slabs" of volume $4dx$ and weight density 9800 that is pumped from a depth of x. So $W = \int_0^2 39{,}200x\,dx = 39{,}200x^2\Big]_0^2 = 78{,}400$ b)

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7. One application of integration by parts to $\int x^3 e^{2x}\,dx$ can reduce it to which of the following?

a) $\frac{1}{2}x^3e^{2x} - \frac{3}{2}\int x^2e^{2x}\,dx$

b) $\frac{1}{2}x^3e^{2x} + \frac{3}{2}\int x^2e^{2x}\,dx$

c) $\frac{1}{2}x^3e^{2x} - 3\int x^2e^{2x}\,dx$

d) $\frac{1}{2}x^3e^{2x} + 3\int x^2e^{2x}\,dx$

e) None of the above

$u = x^3$	$v = e^{2x}/2$
$du = 3x^2dx$	$dv = e^{2x}dx$

So, $\int x^3e^{2x}\,dx = \int u\,dv = uv - \int v\,du = \frac{1}{2}x^3e^{2x} - \frac{3}{2}\int x^2e^{2x}\,dx$. a)

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8. Using the trig substitution $x = 3\sin(\theta)$ changes $\int_0^{3/2} \frac{x^2}{\sqrt{9-x^2}}dx$ to which of the following?

a) $9\int_0^{\pi/3} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

b) $9\int_0^{\pi/6} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

c) $9\int_0^{3/2} \frac{\sin^2(\theta)}{\cos(\theta)}d\theta$

d) $9\int_0^{\pi/3} \sin^2(\theta)d\theta$

e) $9\int_0^{\pi/6} \sin^2(\theta)d\theta$

Note that for $x = 0$, $\sin(\theta) = 0$ so $\theta = 0$, and for $x = 3/2$, $\sin(\theta) = 1/2$ so $\theta = \pi/6$. Thus we get e) since

$$\int_0^{3/2} \frac{x^2}{\sqrt{9-x^2}}dx = \int_0^{\pi/6} \frac{9\sin^2(\theta)}{\sqrt{9-9\sin^2(\theta)}}3\cos(\theta)\,d\theta = \int_0^{\pi/6} \frac{9\sin^2(\theta)}{3\cos(\theta)}3\cos(\theta)\,d\theta = 9\int_0^{\pi/6} \sin^2(\theta)d\theta$$

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9. Noting that $\int e^{-x}(\sin(x)+\cos(x))dx = -e^{-x}\cos(x) + C$, the improper integral $\int_0^{\infty} e^{-x}(\sin(x)+\cos(x))dx$ equals:

a) $\lim_{T\to 0^+} (-e^{-T}\cos(T)+1)$

b) $\lim_{T\to 0^-} (-e^{-T}\cos(T)+1)$

c) $\lim_{T\to\infty} (-e^{-T}\cos(T)+1)$

d) $\lim_{T\to\infty} (-e^{-T}\cos(T))$

e) None of the above

To evaluate such an improper integral, we evaluate at T and 1 and subtract and then take this limit.

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10. If a lamina has mass and moments given by: $m = 3,\ M_x = 5,\ M_y = 7$, then the center of mass, $(\bar{x}, \bar{y})$, equals:

a) $\left(\frac{3}{5}, \frac{3}{7}\right)$

b) $\left(\frac{3}{7}, \frac{3}{5}\right)$

c) $\left(\frac{5}{3}, \frac{7}{3}\right)$

d) $\left(\frac{7}{3}, \frac{5}{3}\right)$

e) None of the above

Since $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(\frac{7}{3}, \frac{5}{3}\right)$ we get d).

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11. The function given by $y = 3\sin(2x)$ is a solution to which of the following second order initial-value problems?

a) $y'' = -y$, $y(0) = 0, y'(0) = 0$

b) $y'' = 4y$, $y(0) = 0, y'(0) = 6$

c) $y'' = -y$, $y(0) = 0, y'(0) = 3$

d) $y'' = -3y$, $y(0) = 0, y'(0) = 6$

e) $y'' = -4y$, $y(0) = 0, y'(0) = 6$

Since $y' = 6\cos(2x)$ and $y'' = -12\sin(2x)$ we see that e) is the answer. The initial values check too.

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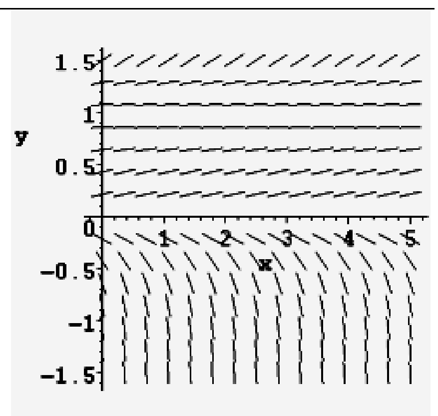
12. To the right is the direction field for differential equation $y' = y(y-1)^2$. For which one of the initial values $y(0) = y_0$ listed below will the limiting behavior of the solutions approach 1 (so that $\lim_{x\to\infty} y(x) = 1$)?



a) $y_0 = 1.5$

b) $y_0 = 0.5$

c) $y_0 = 0$

d) $y_0 = -0.5$

e) $y_0 = -1.0$

For any starting point between 0 and 1 on the y-axis, we see that the "flow" will be toward $y = 1$. Starting above 1 goes to infinity and below 0 to minus infinity. At 0 stays at 0. So b).

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13. A certain isotope decays so that the remaining mass after t days is given by $m = m_0 e^{-0.12t}$ where m_0 is the initial mass. The half life is closest to which of the following?

a) 0.12 days

b) 0.83 days
c) 2.3 days
d) 5.8 days
e) 8.3 days

The half-life will be the time t that satisfies $\frac{1}{2}m_0 = m_0 e^{-0.12t}$. To solve for t we divide by m_0, take the natural logarithm, and divide by -0.12 getting $t = \ln(1/2)/(-0.12) \approx 5.8$ so d).

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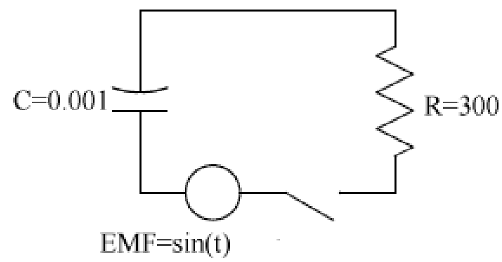
14. Once the switch is closed, a differential equation that describes the circuit to the right is



a) $1000Q' + 300Q = \sin(t)$
b) $300Q' + 1000Q = \sin(t)$
c) $Q'/1000 + 300Q = \sin(t)$
d) $Q'/300 + 1000Q = \sin(t)$
e) $Q'/300 + Q/1000 = \sin(t)$

The voltage drop across the capacitor is $E_C = Q/C = 1000Q$ and across the resistor is $E_R = IR = 300I$. Since $E_R + E_C = EMF$ and $I = Q'$ we have $300Q' + 1000Q = \sin(t)$ or b)

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15. The point with Cartesian coordinates $(x, y) = (2,2)$ can have which of the following for polar coordinates (r,θ)?

a) $(2,\pi/4)$
b) $(2,-\pi/4)$
c) $(2\sqrt{2},-\pi/4)$
d) $(2\sqrt{2},\pi/4)$
e) $(4\sqrt{2},-\pi/4)$

Forming the right triangle with Cartesian coordinates (2,2), (0,0), and (2,0), we see that the hypotenuse is $r = 2\sqrt{2}$ and the angle at the origin is $\theta = \pi/4$. So d).

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16. Find the limit of the sequence given by $a_n = \dfrac{3-5n^2}{2n+n^2}$

a) $-\infty$
b) -5
c) $-5/2$
d) $3/2$
e) $5/2$

Dividing top and bottom by n^2 as using the fact that the limit of $\{1/n\}$ is 0, we have

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{3}{n^2} - 5}{\frac{2}{n} + 1} = \frac{-5}{1} = -5 \text{ so b).}$$

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17. The 5th degree Taylor polynomial, $T_5(x)$, at $a = 0$ (also called a Maclaurin polynomial) for the sine function, $f(x) = \sin(x)$, is:

a) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$

b) $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120}$

c) $1 - \frac{x^2}{2} + \frac{x^4}{24}$

d) $-x + \frac{x^3}{6} - \frac{x^5}{120}$

$\boxed{\text{e) } x - \frac{x^3}{6} + \frac{x^5}{120}}$

Since $\sin(0) = 0$, $\sin'(0) = \cos(0) = 1$, $\sin''(0) = -\sin(0) = 0$, $\sin^{(3)}(0) = -\cos(0) = -1$, $\sin^{(4)}(0) = \sin(0) = 0$, $\sin^{(5)}(0) = \cos(0) = 1$, we have

$T_n(x) = \sin(0) + \frac{\sin'(0)}{1!}x + \frac{\sin''(0)}{2!}x^2 + \frac{\sin^{(3)}(0)}{3!}x^3 + \frac{\sin^{(4)}(0)}{4!}x^4 + \frac{\sin^{(5)}(0)}{5!}x^5 = x - \frac{x^3}{6} + \frac{x^5}{120}$ so e)

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18. The sum of vectors $\mathbf{a} = \langle 1,-2,4 \rangle$ and $\mathbf{b} = \langle 0,2,1 \rangle$ in terms of standard basis vectors, is:

a) $-4\mathbf{j} + 4\mathbf{k}$

$\boxed{\text{b) } \mathbf{i} + 5\mathbf{k}}$

c) $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$

d) $\mathbf{i} + 5\mathbf{j}$

e) $\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

Since $\mathbf{a} + \mathbf{b} = \langle 1,-2,4 \rangle + \langle 0,2,1 \rangle = (1,0,5) = \mathbf{i} + 5\mathbf{k}$ so b).

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19. The scalar projection of vector $\mathbf{b} = \langle 3,4 \rangle$ onto vector $\mathbf{a} = \langle 5,12 \rangle$ is:

a) 63

b) $63/5$

$\boxed{\text{c) } 63/13}$

d) $5/63$

e) $13/63$

The scalar projection is given by $\mathbf{b} \cdot \frac{\mathbf{a}}{|\mathbf{a}|} = \langle 3,4 \rangle \cdot \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle = \frac{15}{13} + \frac{48}{13} = \frac{63}{13}$ so c).

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20. The cross product $\mathbf{k} \times (2\mathbf{j})$ equals:

a) $2\mathbf{i}$

$\boxed{\text{b) } -2\mathbf{i}}$

c) $2\mathbf{j} + \mathbf{k}$

d) $2\mathbf{j} - \mathbf{k}$

e) the zero vector

We have $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ (using the fact that the area the square determined by $\mathbf{k}$ and $\mathbf{j}$ is 1 and the direction perpendicular to both is along the *x*-axis with the negative sign following from the right hand rule. So $\mathbf{k} \times (2\mathbf{j}) = 2(\mathbf{k} \times \mathbf{j}) = -2\mathbf{i}$ or b).

PART TWO: FREE RESPONSE (50%). The remaining 10 problems are not multiple choice. Answer them on this test paper in the blank space provided. If space is insufficient use the backs of the pages. Show the details of your work and $\boxed{\text{box}}$ your answers.

21. Find the volume created by rotating the region bounded by $y = x$ and $y = x^2$ about the *y*-axis both by the method of washers and the method of cylindrical shells (showing that you get the same value).

a) Washers: $V = \int_0^1 \left(\pi \left(\sqrt{y} \right)^2 - \pi y^2 \right) dy = \pi \int_0^1 \left(y - y^2 \right) dy = \boxed{\pi / 6}$

b) Cyl. shells $V=\int_0^1 2\pi x\left(x-x^2\right)dx=2\pi\int_0^1\left(x^2-x^3\right)dx=2\pi\left(\tfrac{1}{3}x^3-\tfrac{1}{4}x^4\right)\Big]_0^1=2\pi\tfrac{1}{12}=\boxed{\pi/6}$

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22. Suppose the work required to stretch a spring 2 ft beyond its natural length is 12 ft-lb.

a) Find the spring constant k for this spring.

By Hooke's law, force is given by $f(x)=kx$, so work is given by the integral and

$12=\int_0^2 kx\,dx=kx^2/2\Big]_0^2=2k$, so $\boxed{k=6}$.

b) How much work is needed to stretch a spring with the same spring constant 3 ft beyond its natural length?

$W=\int_0^3 6x\,dx=3x^2\Big]_0^3=\boxed{27\text{ ft-lb}}$.

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23. a) Find constants A and B so that $\dfrac{12}{x(x+2)}=\dfrac{A}{x}+\dfrac{B}{x+2}$.

Multiplying by $x(x+2)$ gives $12=A(x+2)+Bx=(A+B)x+2A$. So $A+B=0$ and $2A=12$. Thus $A=6$ while $B=-6$.

b) Evaluate by hand $\int\dfrac{12}{x(x+2)}dx$ using part a), showing all work.

$$\int\frac{12}{x(x+2)}dx=6\int\frac{1}{x}dx-6\int\frac{1}{x+2}dx=6\ln(x)-6\ln(x+2)+C=\boxed{6\ln\left(\frac{x}{x+2}\right)+C}$$

c) Consider the series $\sum_{n=1}^{\infty}\dfrac{12}{n(n+2)}$. Express the eighth partial sum, s_8, as a fraction in lowest terms. (Hint: use part a.)

As in a, $\dfrac{12}{n(n+2)}=\dfrac{6}{n}-\dfrac{6}{n+2}$. So the first 8 terms will give us (canceling most terms)

$$s_8=\left(\frac{6}{1}-\frac{6}{3}\right)+\left(\frac{6}{2}-\frac{6}{4}\right)+\left(\frac{6}{3}-\frac{6}{5}\right)+\left(\frac{6}{4}-\frac{6}{6}\right)+\left(\frac{6}{5}-\frac{6}{7}\right)+\left(\frac{6}{6}-\frac{6}{8}\right)+\left(\frac{6}{7}-\frac{6}{9}\right)+\left(\frac{6}{8}-\frac{6}{10}\right)=.$$

$$\frac{6}{1}+\frac{6}{2}-\frac{6}{9}-\frac{6}{10}=\frac{696}{90}=\frac{116}{15}$$

d) Find the sum of the series in part c by taking the limit of the partial sums.

By analogy with part c,

$s_n=\dfrac{6}{1}+\dfrac{6}{2}-\dfrac{6}{n+1}-\dfrac{6}{n+2}$. So $\sum_{n=1}^{\infty}\dfrac{12}{n(n+2)}=\lim\limits_{n\to\infty}s_n=\dfrac{6}{1}+\dfrac{6}{2}=\boxed{9}$

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24. a) Use substitution to show that the trigonometric integral $\int_0^{\pi}\sin(x)\cos^2(x)\,dx$ equals 2/3.

Letting $u=\cos(x)$ we have $du=-\sin(x)\,dx$, so $\int_0^{\pi}\sin(x)\cos^2(x)\,dx=\int_{\bullet}^{\bullet}-u^2du=-\tfrac{1}{3}u^3\Big]_{\bullet}^{\bullet}=$

$-\tfrac{1}{3}\cos^3(x)\Big]_0^{\pi}=\tfrac{1}{3}-\left(-\tfrac{1}{3}\right)=2/3$.

b) Use Simpson's rule with 4 subintervals to approximate the integral in part a. (Give the answer to 6 decimal places.)

The interval endpoints are at 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π so applying the integrand to these points we get

$y_0=0$, $y_1=\dfrac{1}{2\sqrt{2}}$, $y_2=0$, $y_3=\dfrac{1}{2\sqrt{2}}$, $y_4=0$. So $S_4=\dfrac{\Delta x}{3}\left(y_0+4y_1+2y_2+4y_3+y_4\right)=$

$\frac{\pi}{12}\cdot\frac{8}{2\sqrt{2}} = \boxed{\frac{\pi\sqrt{2}}{6} \approx 0.740480}$.

25. a) Solve the initial-value problem: $\frac{dy}{dt} = te^y$, $y(0)=0$, giving an explicit solution for y as a function of t.

Separating variables, $e^{-y}dy = t\,dt$ and integrating gives $-e^{-y} = \frac{t^2}{2}+C$. From the initial value, for $t=0$ we have $y=0$ so $C=-1$. So solving for y, we get: $e^{-y} = 1-\frac{t^2}{2}$, then $-y = \ln\left(1-\frac{t^2}{2}\right)$, and $\boxed{y = -\ln\left(1-\frac{t^2}{2}\right)}$.

b) Use Euler's method with $n=2$ steps to approximate $y(1)$ in the problem in part a).

We fill in the following table, using $h=0.5$.

kth step	t_k	y_k	$F(t_k,y_k)=t_k e^{y_k}$	$\Delta y = hF(t_k,y_k)$	$y_{k+1}=y_k+\Delta y$
0	0	0	0	0	0
1	0.5	0	0.5	0.25	0.25
2	1	0.25			

So we conclude $\boxed{y(1)\approx 0.25}$. By comparison, $y(1) = -\ln(1/2) \approx 0.693147$

26. An R-L circuit with a 2 henry inductor, a 20 ohm resistor, and an EMF of $E(t)=e^{-5t}$ leads to the differential equation for current: $2I' + 20I = e^{-5t}$. Assuming a switch closes the circuit at time 0 (so that $I(0)=0$), solve for $I(t)$.

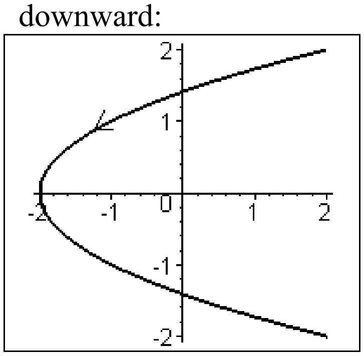
We divide by 2 to get $I' + 10I = \frac{1}{2}e^{-5t}$ so the integrating factor is $\mu(t) = e^{10t}$. Hence $\left(Ie^{10t}\right)' = \frac{1}{2}e^{5t}$. Integrating both sides gives $Ie^{10t} = \frac{1}{10}e^{5t} + C$. Since $I(0)=0$, $C=-1/10$.

So $\boxed{I(t) = \frac{1}{10}e^{-5t} - \frac{1}{10}e^{-10t}}$.

27. a) Eliminate the parameter to find a (simplified) Cartesian equation for the parametric curve $x = 4t^2 - 16t + 14$, $y = 4-2t$ and use it to sketch the curve.

Since $t = 2-\frac{1}{2}y$, $x = 4\left(2-\frac{1}{2}y\right)^2 - 16\left(2-\frac{1}{2}y\right) + 14 = 4\left(4-2y+\frac{1}{4}y^2\right) - 32 + 8y + 14 = y^2 - 2$.

We get a parabola "on its side". Since y decreases as t increases, the arrow on the curve goes downward:



b) Use the parametric equations in part a) to verify that the tangent line to the curve at $t=1$ has equation $x - 4y + 6 = 0$

Since $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2}{8t-16}$ we have at $t=1$ the slope is $\frac{-2}{-8} = \frac{1}{4}$. Also for $t=1$, $(x,y) = (2,2)$, so the tangent line is given by $y - 2 = \frac{1}{4}(x-2)$ which simplifies to $x - 4y + 6 = 0$.

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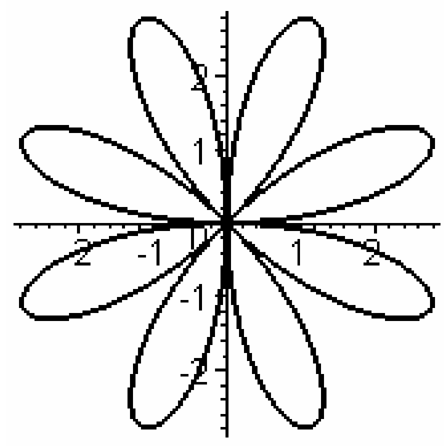
28. Find the area of one "petal" of the graph in polar coordinates of $r = 3\sin(4\theta)$ shown to the right.



First we note that for one petal we can take θ as going from 0 to $\pi/4$. So $A = \int_0^{\pi/4} \frac{1}{2} r^2 d\theta = \frac{9}{2}\int_0^{\pi/4} \sin^2(4\theta) d\theta =$

$\frac{9}{4}\int_0^{\pi/4} (1-\cos(8\theta))d\theta = \frac{9}{4}\left(\theta - \frac{1}{8}\sin(8\theta)\right)\Big]_0^{\pi/4} = \frac{9}{4}\cdot\frac{\pi}{4} =$

$\boxed{\frac{9\pi}{16} \approx 1.76715}$

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29. a) Find the unit vector in the direction from point $(2,-1,4)$ to $(3,-2,6)$.

Subtracting initial point from terminal point we get $\langle 1,-1,2\rangle$. The length of this vector is $|\langle 1,-1,2\rangle| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$ so the unit vector is $\boxed{\left\langle \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle}$.

b) Show that the vector found in part a) is orthogonal (perpendicular) to $2\mathbf{i} - \mathbf{k}$.

Since the dot product, $\left\langle \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle \cdot \langle 2,0,-1\rangle = \frac{2}{\sqrt{6}} + 0 - \frac{2}{\sqrt{6}} = 0$ we know they are orthogonal.

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30. Note that the lines with parametric equations $x = 2t,\ y = 1+t,\ z = -3-t$ and $x = 2-t,\ y = 2+t,\ z = -4+2t$ both pass through the point $(2,2,-4)$. Find an equation for the plane that contains both lines.

To get the normal direction of the plane we take the cross product of the directions of the lines: $\langle 2,1,-1\rangle \times \langle -1,1,2\rangle = \langle 3,-3,3\rangle$. So an equation is $3(x-2) - 3(y-2) + 3(z+4) = 0$ which can be rewritten as $\boxed{x - y + z + 4 = 0}$

(end)