# Calculus II    Chapters 5 & 6    Practice Problems
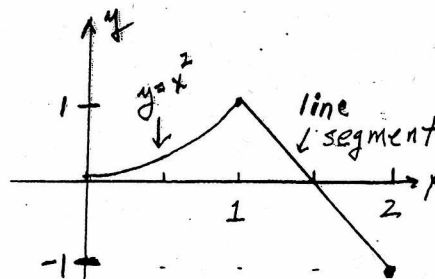
**Name** _____

1. Consider the function $f$ graphed on the right.

(a) Find the exact value for $\int_0^2 f(x)\,dx$.

(b) Approximate $\int_0^2 f(x)\,dx$ using $R_4$ (the right endpoint rule using 4 subdivisions) and show the rectangles you are using on the graph.

(c) Approximate $\int_0^2 f(x)\,dx$ using $M_2$ (the midpoint rule using 2 subdivisions).

Solve problems 2 - 5 by hand. Show every step. Use your calculator only to check your answers.

2.  (a) $\int \dfrac{x^3}{\sqrt{x}}\,dx$,   (b) $\int e^x \cos(e^x)\,dx$,   (c) $\int_0^1 x\sqrt{3x^2+1}\,dx$,   (d) $\dfrac{d}{dx}\int_2^x t\,\sin(t^3)\,dt$.

3. Sketch both $y=\sin(x)$ and $y=\cos(x)$ over the interval $[0,\dfrac{5\pi}{4}]$. Find the area bounded between the curves over that interval.

4. Sketch and find the area of the region bounded by the curves $x=y^2-2$ and $y=x$.

5. (a) State the Mean Value Theorem for Integrals and prove by applying the Mean Value Theorem for derivatives to $F(x)=\int_a^x f(t)\,dt$. See problem #23 p.445 of our text book.

(b) For $f(x)=x^2$ on $[-1,2]$, find the number $c$ where $f_{ave}=f(c)$.

(c) Sketch the graph of $f$ and a rectangle whose area is the same as the area under the graph of $f$.

For the remaining problems, set up the necessary integrals and use your calculator to evaluate them.

6. Let R be the region under the curve $y=\sin(x)$ over the interval $[0,\pi]$. Find the volume generated by revolving R about the axis $y=-1$.

7. A force of 30 N is required to hold a spring that has been stretched from its natural length of 20 cm to a length of 25 cm. How much work is done in stretching the spring from 25 to 30 cm?

8. A tank full of water has the shape of a paraboloid of revolution (see figure). Find the work required to pump the water out of the top of the tank. Use the fact that water weights 62.5 lbs/ft³.
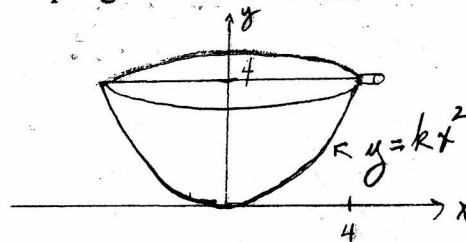
Short answers: (1a) 1/3, (1b) .125, (1c) .25; (2a) $\dfrac{2}{7}x^{7/2}+C$, (2b) $\sin(e^x)+C$; (2c) 7/9; (2d) $x\sin(x^3)$; (3) $3\sqrt{2}-1$; (4) 9/2; (5b) $c=-1$ or 1; (6a) $\pi(\pi+8)/2$, (6b) $2\pi(8-\pi)$; (7) 2.25 J; (8) $8000\pi/3\ ft-lb$