Calculus II Chapters 7 - 9 Practice Problems

Name ___________________________ Spring 2011

1. Evaluate the following integrals by hand. Show all steps. Use your calculator to check your answers:

(a) $\int x\sin(2x)\ dx$ (b) $\int x^6\ln(x)\ dx$ (c) $\int x^2 \arctan(x)\ dx$

(d) $\int \frac{4x+3}{x^2+3x+2}dx$ (e) $\int \frac{3x^2+x+3}{x^3+x}dx$ .

2. Simpson's rule uses parabolas to approximate integrals. It is not surprising that it gives the exact result when integrating 2nd degree polynomials. Remarkably, it also gives the exact answer when integrating polynomials of degree three.

(a) Evaluate $\int_0^4 x^3 dx$. (b) Use $S_4$, Simpson's Rule to approximate the integral in (a).

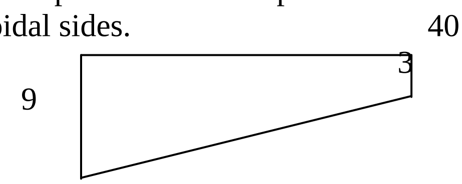
(c) Use $T_4$, the Trapezoidal Rule to approximate the integral in (a).

3. Assume that $f$ is a function defined on the interval [a , b] and $f' < 0$ and $f'' > 0$ on [a , b]. Rank the following from least to greatest: $L_n$, $R_n$, $M_n$, $T_n$, and $\int_a^b f(x)dx$, where $L_n$, $R_n$, $M_n$, $T_n$ are the left, right, midpoint, and trapezoidal rules respectively using n subintervals. Draw figures to justify your answer.

4. Evaluate the following improper integrals by hand. Show all steps. Sketch the curve and the region represented by the integral. Use your calculator to check your answers:

(a) $\int_1^\infty \frac{1}{\sqrt{x}}dx$ (b) $\int_0^1 \ln(x)\ dx$ .

5. A swimming pool is 20 ft wide and 40 ft long and its bottom is an inclined plane. The shallow end has a depth of 3 ft and the deep end 9 ft. If the pool is full of water, find the hydrostatic force on one of the trapezoidal sides.



6. Find the centroid $(\bar{x}, \bar{y})$ for the right triangle with vertices at ( 0 , 0 ), ( a , 0 ), and ( a , b ). Estimate $(\bar{x}, \bar{y})$ first.

7. Find the solution to the initial value problem $\frac{dy}{dx} = \frac{x + \sin(x)}{3y^2}$ , $y(0) = 2$

8. Answer the questions below for the differential equation $y' = 3 - y$ .

(a) Draw the direction field for the differential equation.

(b) Use your direction field to draw the solution going through the origin.

(c) If $y(0) = 0$, use two steps of Euler's method to approximate $y(.2)$ .

9. If $y(t)$ is the temperature of an object in a room of constant temperature $R$ , then the differential equation for Newton's Law of Cooling is $y'(t) = k(y(t) - R)$ .

a) Explain what this differential equation means in plain English.

b) A can of soda has temperature $40^o F$ when taken from a cooler and placed in a room of $72^o F$ . Five minutes later the soda is $56^o F$ . When will the temperature of the soda be $64^o F$ ?

10. A simple series circuit consists of a 25 ohm resistor, a .01 farad capacitor and a constant EMF $E(t) = 50$ volts. If the initial charge on the capacitor is 4 coulombs, set up and solve an initial value problem to determine the charge $Q(t)$ and current $I(t)$ for $t > 0$ .

Short answers: (1) use calculator to verify answers to integrals; (2) (a) = (b) = 64, (c) = 68 ; (3) $R_n < M_n < \int_a^b f(x)dx < T_n < L_n$ ; (4a) diverges to $\infty$, (4b) converges to -1; (5) 48,750 lbs; (6) ( 2a/3 , b/3 ); (7) $y = \sqrt[3]{x^2/2 - \cos(x) + 9}$ ; (8c) $y(.2) \approx .57$ ; (9) t = 10 mins; (10) $Q(t) = .5 + 3.5e^{-4t}$ , $I(t) = -14e^{-4t}$ .