Score:

Name:_____

Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM122 – Test #4– ~~Fall~~ Spring 2010

Closed book. Calculators/one note sheet allowed. Properly label all graphs. Box/circle your final answer. YOU MUST SHOW ALL WORK FOR FULL CREDIT.

1. (20 points) Given the following points: P (1, 0, -2), Q (1, 1, 1), R (0, -3, -1), complete the table below (do not use calculator):

$\overrightarrow{PQ}$	$\langle 0,1,3\rangle$	$\lvert\overrightarrow{PQ}\rvert$	$\sqrt{10}$	$\overrightarrow{PQ}\cdot\overrightarrow{PR}$	0
$\overrightarrow{QP}$	$\langle 0,-1,-3\rangle$	$\lvert\overrightarrow{PR}\rvert$	$\sqrt{11}$	$\overrightarrow{QP}\cdot\overrightarrow{QR}$	10
$\overrightarrow{PR}$	$\langle -1,-3,1\rangle$	$\lvert\overrightarrow{RQ}\rvert$	$\sqrt{21}$	$\overrightarrow{RQ}\cdot\overrightarrow{RP}$	11
$\overrightarrow{RP}$	$\langle 1,3,-1\rangle$	$2\overrightarrow{PQ}$	$\langle 0,2,6\rangle$	$\overrightarrow{PQ}\times\overrightarrow{PR}$	$\langle 10,-3,1\rangle$
$\overrightarrow{QR}$	$\langle -1,-4,-2\rangle$	$\overrightarrow{QP}+\overrightarrow{QR}$	$\langle -1,-5,-5\rangle$	$\overrightarrow{QP}\times\overrightarrow{QR}$	$\langle -10,3,-1\rangle$
$\overrightarrow{RQ}$	$\langle 1,4,2\rangle$	$3\overrightarrow{QR}+4\overrightarrow{RP}$	$\langle 1,0,-10\rangle$	$\overrightarrow{RQ}\times\overrightarrow{RP}$	$\langle -10,+3,-1\rangle$

Fill in answers carefully! Highlighted data in this table can be used in problems 2-4.

$+\langle -3,-12,-6\rangle$
$+\langle 4,12,-4\rangle$
$\langle 1,0,-10\rangle$

$\overrightarrow{QP} \times \overrightarrow{QR}$: $\begin{vmatrix} i & j & k \\ 0 & -1 & -3 \\ -1 & -4 & -2 \end{vmatrix}$ $\langle 10,3,-1\rangle$

$\overrightarrow{RQ} \times \overrightarrow{RP}$: $\begin{vmatrix} i & j & k \\ 1 & 4 & 2 \\ 1 & 3 & -1 \end{vmatrix}$ $\langle -10,+3,-1\rangle$

No marks on this table		
MC (10 pts)		
GW (10 pts)		
1 (20 pts)		
2 (20 pts)		
3 (10 pts)		
4 (10 pts)		
5 (10 pts)		
6 (10 pts)		

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2. Answer the following:

a. (5 points) Find the angle between the vectors $\overline{QR}$ and $\overline{QP}$?

$$\cos^{-1}\left(\frac{\vec{QR}\cdot\vec{QP}}{|\vec{QR}||\vec{QP}|}\right)=\cos^{-1}\left(\frac{10}{\sqrt{210}}\right)=\boxed{46.36^\circ}$$

b. (5 points) Find the area of triangle PQR using cross products.

$$A=\frac{1}{2}\left|\vec{PQ}\times\vec{PR}\right|=\frac{1}{2}\left(10^2+(-3)^2+1^2\right)$$

$$=\boxed{\frac{1}{2}\sqrt{110}\approx 5.24}$$

c. (5 points) Calculate $\overline{PQ}\cdot\left(\overline{PQ}\times\overline{PR}\right)$ and $\overline{PR}\cdot\left(\overline{PQ}\times\overline{PR}\right)$. Explain your results.

Both are zero because $\vec{PQ}\times\vec{PR}$ is perpendicular to both $\vec{PQ}$ & $\vec{PR}$

d. (5 points) Is triangle PQR a right triangle? Why?

Yes

① $\vec{PQ}\cdot\vec{PR}=0$ indicating a right angle

② $|\vec{PQ}|^2+|\vec{PR}|^2=|\vec{RQ}|^2$ true only for right triangles

3. Find:

a. (5 points) The vector equation of line connecting points P and Q.

$\langle x, y, z \rangle = \langle 1, 0, -2 \rangle + \langle 0, 1, 3 \rangle t$

$\langle 1, 0, -2 \rangle$: initial point P; $\langle 0, 1, 3 \rangle$: direction vector $\vec{PQ}$

b. (5 points) The equation of a plane containing points P, Q and R (write equation in form $ax + by + cz = d$)

use $\vec{PQ} \times \vec{PR} = \langle 10, -3, 1 \rangle$ & $Q(1, 1, 1)$

(any cross product or point will work)

$\Rightarrow 10(x-1) - 3(y-1) + (z-1) = 0$

$\Rightarrow \boxed{10x - 3y + z = 8}$

4. Find the following:

a. (5 points) The scalar projection of $\overrightarrow{QR}$ onto $\overrightarrow{QP}$ $\left(comp_{\overrightarrow{QP}}\overrightarrow{QR}\right)$.

$= \dfrac{\vec{QR} \cdot \vec{QP}}{|\vec{QP}|} = \dfrac{10}{\sqrt{10}} = \boxed{\sqrt{10}}$

b. (5 points) The vector projection of $\overrightarrow{QP}$ onto $\overrightarrow{QR}$ $\left(proj_{\overrightarrow{QR}}\overrightarrow{QP}\right)$

$\dfrac{\vec{QP} \cdot \vec{QR}}{|\vec{QR}|^2} \vec{QR} = \boxed{\dfrac{10}{21} \langle -1, -4, -2 \rangle}$

$= \boxed{\left\langle \dfrac{-10}{21}, \dfrac{-40}{21}, \dfrac{-20}{21} \right\rangle}$

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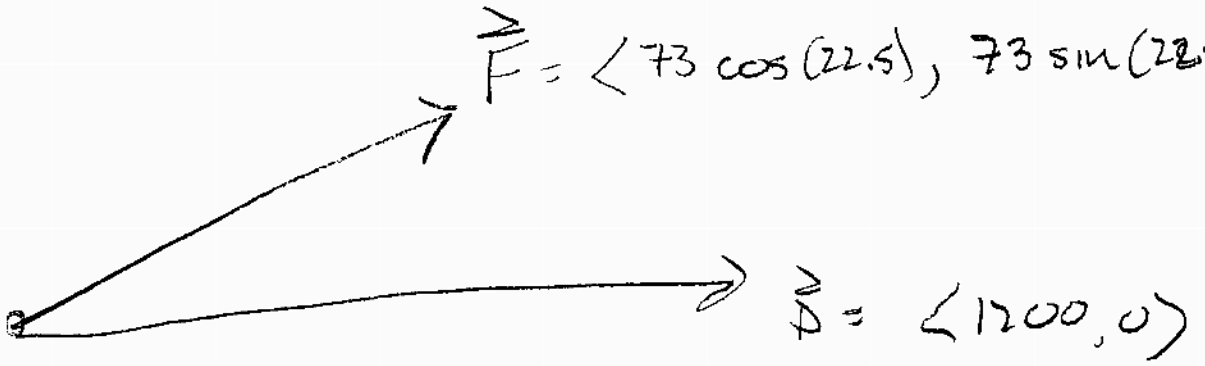
5. (10 points) Somewhere on the Severn River: A YP sets up a tow line to rescue a floundering sailboat. The towline forms a 22.5 degree angle with the water line. A total of 73 lbs of force is required to tow the sail boat. How much work is done in towing the vessel 1200 feet?



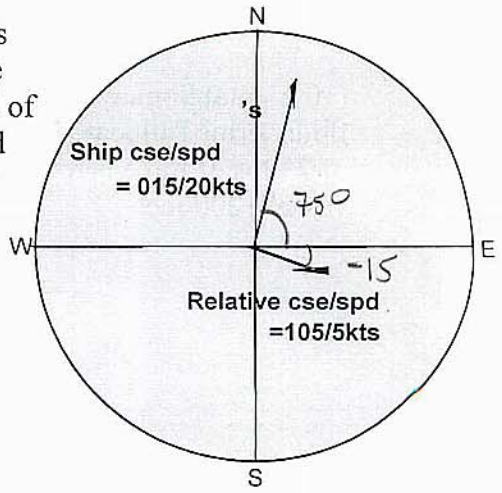
$W = \vec{F} \cdot \vec{D} = 73\cos(22.5)(1200) + 73\sin(22.5)(0)$

$\Rightarrow 0$

$= \boxed{80931.8 \text{ ft-lbs}}$

6. (10 points) Somewhere in the South Pacific: Your ship is traveling on a course 015 at a speed of 20 knots. You are tracking a contact whose relative course is 105 at a speed of 5 knots. Use vector addition to find the true course/speed of the contact.

Hint: $\overrightarrow{True\ Cse} = \overrightarrow{Ship\ Cse} + \overrightarrow{Relative\ Cse}$



SHIP = $\langle 20\cos(75), 20\sin(75)\rangle$

+ REL = $\langle 5\cos(-15), 5\sin(-15)\rangle$

True = $\langle 10.00, 18.02\rangle$

magnitude = $(10.00^2 + 18.02^2)^{1/2} = 20.61$ kts

direction = $\tan^{-1}\left(\frac{18.02}{10.00}\right) = 60.97°$

↑ convert to course

i.e $90 - 60.97$

⇒ **True = 20.61 kts at 29.02°**